Towards Realistic Prosthetic Gait Simulations: Enhancing the Accuracy of OpenSim Analysis by Integrating the Transfemoral Prosthesis Model

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Abstract—Powered transfemoral prostheses offer the potential to improve mobility and quality of life for individuals with amputations. This study aimed to develop and validate an OpenSim model of a subject with a unilateral transfemoral amputation wearing a powered transfemoral prosthesis and to compare the model's performance with that of a model without prosthesis characteristics. We utilized experimental walking data from a single transfemoral amputee subject to demonstrate the feasibility of the model. Inverse kinematics and inverse dynamics were performed to compare the results using the encoder and current data of the knee and ankle actuators, which served as ground truth. The model with prosthesis characteristics demonstrated a closer match to the actuator data, particularly during the stance phase, suggesting that it better reflects the dynamic features of a real powered prosthesis. However, discrepancies were observed during the swing phase, highlighting the need for further refinements. This study provides valuable insights into the importance of incorporating prosthesis characteristics in biomechanical models to simulate joint behavior accurately. It has implications for the development and assessment of prosthetic devices.

I. INTRODUCTION

The field of powered lower limb research is dedicated to enhancing mobility for individuals with amputations. In the United States alone, the number of lower extremity amputees is increasing by roughly 150,000 annually [1]. Remarkable advancements in prosthetic technology have greatly enhanced the support available to lower extremity amputees, offering improved functionality and a promising increase in their quality of life [2]. Compared to passive prostheses, powered lower limb prostheses are able to deliver energy, resulting in greater stability and mobility. Additionally, these prostheses are designed to replicate natural gait patterns, leading to increased user satisfaction [3]. So, powered lower limb prostheses have the potential to improve social integration and independence for users and are currently the focus of extensive research efforts [4], [5].

Biomechanical analysis of gait is a valuable tool for understanding human walking patterns. By examining the movement of the body during walking, this analysis can uncover deviations and anomalies in an individual's gait,

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such as asymmetry or muscle imbalances. This type of analysis is particularly useful for individuals with prosthetic limbs, as it can help evaluate the efficacy of these devices and identify potential risks associated with their use [6], [7]. Additionally, biomechanical analysis can be used to assess the success of rehabilitation programs for prosthetic users [8]. Ultimately, the insights gained from such analyses can help improve mobility and quality of life for individuals with prosthetic limbs.

Musculoskeletal modeling and dynamics simulation have emerged as powerful tools used to simulate and analyze human locomotion. These are carried out by leveraging motion capture, force platform, electromyography (EMG), Inertial Measurement Unit (IMU) data, and more. The opensource software OpenSim allows researchers and clinicians to create customized musculoskeletal models for a variety of applications, including analyzing movement patterns, designing orthopedic interventions, and developing prosthetic devices [9], [10]. In addition to models that analyze the locomotion of the upper extremity and the mechanics of the lumbar spine, OpenSim has wearable robot models such as exoskeletons and prostheses. While there are many models and studies of exoskeletons [11], [12], [13], biomechanical analysis of prosthetic users' gait using musculoskeletal model, especially powered transfemoral prostheses, are rare [14]. This can be due to the different levels of amputation and the need to adjust the prosthetic device for individual users, as well as the costly and time-consuming process of tuning prosthetics for customizing.

In the field of wearable robotics research, numerous studies have been conducted on the development and implementation of powered transfemoral prostheses. These investigations have focused on estimating the gait phase in order to better integrate the amputee patients' intent during movement [15], [16], [17], [18], [19]. To more accurately emulate the complex anatomical structure of the human foot, researchers have explored various foot and toe designs [20], [21]. Additionally, control frameworks employing trajectory tracking and impedance control schemes have been examined in an effort to replicate natural human walking while addressing contact constraints and navigating unknown terrains [7], [22], [23], [24]. The majority of these investigations can greatly benefit from the advanced analytical and forward simulation capabilities provided by OpenSim.

To the authors' knowledge, there are few studies that have created and validated transfemoral amputee models, especially for wearing the powered prosthesis actuated in knee and ankle joints. Raveendranathan et al. developed and

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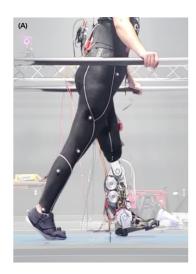
validated a generic musculoskeletal model of an osseointegrated transfemoral amputee [25]. They have successfully obtained a capable tool for studying and better understanding the biomechanics of osseointegrated transfemoral amputees through OpenSim models. However, their validation procedures did not take advantage of the opportunity to directly measure joint states in artificial limbs, which could serve as a source of ground truth data for comparison. Camargo et al. created a model integrating a powered prosthesis, an open-source bionic leg actuated in ankle and knee joints. They validated the model by comparing the results of inverse kinematics using optical motion capture data and encoder sensor data onboard to each joint [14]. However, they only performed validation of their model with a primary focus on kinematic factors, while kinetic factors were not addressed. The lack of consideration for kinetic factors, such as forces and moments acting on the joints, may lead to an incomplete understanding of the biomechanical interactions between the prosthesis leg and the user. This oversight could potentially impact the accuracy and applicability of the model in predicting real-world performance and guiding the design of improved prosthetic limbs.

In this paper, we present the development of an OpenSim musculoskeletal model for a unilateral transfemoral amputee wearing a powered prosthesis. We utilized experimental walking data from a transfemoral amputee subject to demonstrate the feasibility of the model. To obtain simulation results from OpenSim, the model was scaled based on static marker data to reflect the subject's anthropometry, and inverse kinematics and inverse dynamics analyses were conducted for 13 gait trials. We then compared the inverse kinematics and inverse dynamics results using the encoder data from the knee and ankle actuators, which served as the ground truth. To validate the effectiveness of the model wearing the prosthesis, we also performed the same procedure for a model without the prosthesis, aligning the knee and ankle axes. The contribution of this study is to validate the model with prosthesis in Opensim, leveraging experimental gait data. The validation is achieved through inverse kinematics and dynamics analysis, compared with angle and torque values derived from encoder data of the actuators of the powered prosthesis. The effectiveness of the model with the prosthesis is further demonstrated by comparing it to a model without a prosthesis.

II. METHODS

A. Experimental protocols

This study recruited a subject (female, 23 yrs, 164 cm, 66 kg w/o prosthesis) with unilateral transfemoral amputation. Experiments were conducted using a custom-built powered transfemoral prosthesis, AMPRO II [7], [26], that is operated by a microprocessor (BeagleBone Black, Texas Instruments, Dallas, TX, USA) to control the actuated ankle and knee joints (Fig. 1(A)). The actual ankle/knee joint kinematics and kinetics of the prosthesis side were collected at 200 Hz by two high-resolution optical encoders (E5, US Digital, Vancouver, WA, USA) and two motor drivers (Gold-Solo



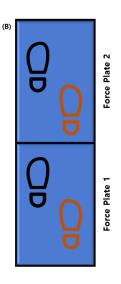


Fig. 1: (A) An amputee participant wears the prosthesis. (B) Two separate force plates built into treadmill

Whistle, Elmo Motion Control, Petach-Tikva, Israel) at each joint. The detailed specifications of the AMPRO II are shown in Table I. All experimental protocols were reviewed and approved by the Institutional Review Board (IRB) at Texas A&M University (IRB2015-0607F), and the participant underwent eight training sessions prior to data collection to acclimate to the powered prosthesis and control framework. Motion data were collected at 100 Hz using a 44-camera Vicon system (Vantage V5, Vicon, Hauppauge, NY, USA) while the participant walked on an instrumented treadmill with two built-in force plates (Tandem, AMTI, Watertown, MA, USA) which were collected at 1000 Hz. To ensure safety and avoid fatigue, the participant maintained a fixed speed of 0.67 m/s during all walking trials. A total of 17 reflective markers (eight on the intact side and nine on the prosthesis side) were utilized to collect three-dimensional trajectories of the lower limbs (Fig. 1)

TABLE I: Specification of AMPRO II

Parameter	AMPRO II	
Total Mass (kg)	4.8	
Peak Torque (Nm)	120	
Knee Range of Motion (deg)	-80 to 5	
Ankle Range of Motion (deg)	-60 to 60	
Height (mm)	345	
Width (mm)	119	

B. Control Framework

AMPRO II employed a hybrid control framework that combines impedance and proportional-derivative (PD) controllers [15], [24]. During the stance phase, the impedance controller generates human-like joint torques for each joint, dependent on the user's walking state as follows [24]:

$$\tau = K \cdot (\theta_{act} - \theta_{ea}) + D \cdot \dot{\theta}_{act} \tag{1}$$

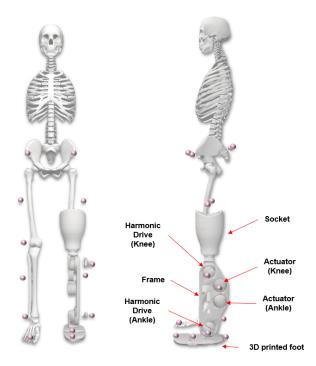


Fig. 2: Generic OpenSim model with unilateral transfemoral amputation. The model includes a bilateral lower limb marker set and defines leg markers that are of easy identification based on the leg landmarks.

where K and D denote the joint stiffness and damping parameters, respectively. θ_{eq} represents the equilibrium angle, while θ_{act} and $\dot{\theta}_{act}$ are the joint position and velocity measured by optical encoders. Continuously varying K and D values were provided for each joint to ensure smooth torques for both the ankle and knee. During the swing phase, when the prosthesis was not in contact with the ground, a PD controller was utilized to follow the desired human-like joint trajectories [22]. This is described by the equation:

$$\tau = K_p \cdot (\theta_{act} - \theta_{des}) + K_d \cdot (\dot{\theta}_{act} - \dot{\theta}_{des})$$
 (2)

where K_p and K_d represent the proportional and derivative gains, respectively. θ_{des} and $\dot{\theta}_{des}$ denote the desired human-like position and velocity for each joint, while θ_{act} and $\dot{\theta}_{act}$ indicate the joint position and velocity. All impedance parameters (i.e., stiffness and damping coefficients) and desired trajectories were provided based on the phase variable to enable synchronized control with the user. It is important to note that all control parameters were initialized at heel-strike to commence the new gait cycle.

C. OpenSim Model

In the construction of a generic model representative of an individual with a unilateral transfemoral amputation, modifications were implemented on the base "Gait2354 Simbody" model, courtesy of OpenSim. In an attempt to isolate the influences of joint torques, muscular components were eliminated from the model. Further simplification was achieved through the fixation of the torso's degrees of freedom (DOF),

thereby enabling the sole observation of the lower limb kinematics. The model's complexity was further reduced by excluding the joints in the feet. The prosthetic component of the model was composed of various parts, including a socket, a knee rotation segment, a frame, an ankle rotation segment, and a foot (Fig. 2). The rotation segments were equipped with actuators and harmonic drives. Flexion-only movement was permitted at the knee and ankle joints, facilitated by their incorporation into the harmonic drives. This resulted in a model with a total of 16 DOFs. A thorough depiction of the effects of a transferoral amputation on the femur was achieved by executing a transaction procedure, resulting in the removal of all body parts distal to the impacted femoral region. This was complemented with subsequent adjustments to mass, inertia, and graphic descriptions to better represent the physical alterations resulting from the amputation.

D. Data Processing

Marker, encoder, and processed ground reaction force (GRF) data were filtered using a third-order Butterworth low-pass filter with 6, 10, and 10 Hz cutoff frequencies, respectively. The C3D file, containing marker and GRF data, was converted to .trc and .mot files using the osimC3D function provided by OpenSim. To perform inverse dynamics in OpenSim, we converted each force and position where the forces are applied on each force plate to forces and positions acting on each foot. Considering the force plates were placed adjacently (Fig. 1(B)), we needed to partition the converted data into distinct GRF data for each foot. The segmentation process was as follows. For a single gait cycle, we segmented it into the following events: prosthesis-side heel strike, intactside toe-off, intact-side heel strike, prosthesis-side toe-off, and the subsequent prosthesis-side heel strike. When a foot was positioned on each of the two force plates, the GRF data were assigned to the respective foot. In cases where a transition occurred between the force plates during singlelimb support, the force was attributed to the foot with the larger force value, and the force location was determined based on the distribution of force across the two force plates. For instances of fluctuations, interpolation techniques were employed to eliminate these fluctuations.

redUtilizing encoders, we were able to obtain angles. By differentiating these angle values with respect to time, we were able to get angular velocity. Further, by applying the known motor gear ratio, the torque sensitivity, and the encoder's current values, we could calculate the torque as ground truth data.

To scale the generic model, we utilized static motion capture data and a marker that was placed on the model (Fig. 2). The marker's relative distances were used to adjust the model dimensions in order to match the size and proportions of the subject's body, excluding the prosthesis parts. The mass and inertia properties of each body are compensated for by the scaling factor. We conducted inverse kinematics and inverse dynamics using scaled models and experimental walking data. Inverse kinematics was performed by minimizing the discrepancy between the predicted marker positions of the

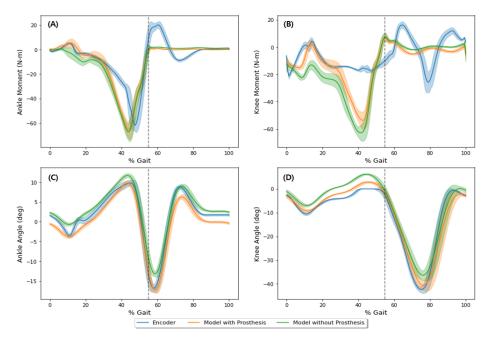


Fig. 3: Prosthesis side joint kinematics/kinetics. Results presented as mean \pm SD (shaded region) of 13 gaits. Blue represents the encoder data, orange corresponds to the model with prosthesis, and green denotes the model without prosthesis. Black dashed line separates the stance phase from the swing phase.

TABLE II: RMSE between ground-truth and the estimated results for the two models and comparison of two models about full phase and stance phase. * indicates statistical significance (p < 0.01).

	RMSE	Ankle Torque	Knee Torque	Ankle Angle	Knee Angle
Full Phase	Model with Prosthesis	$11.425 \pm 1.566^*$	$15.401 \pm 0.911^*$	2.229 ± 0.569	$4.236 \pm 1.696^*$
	Model without Prosthesis	$12.103 \pm 1.749^*$	$19.474 \pm 1.187^*$	2.203 ± 0.458	$5.306 \pm 1.201^*$
Stance Phase	Model with Prosthesis	$12.548 \pm 2.225^*$	$17.646 \pm 1.457^*$	$5.153 \pm 0.242^*$	$1.824 \pm 0.282^*$
	Model without Prosthesis	$13.761 \pm 2.391^*$	$23.768 \pm 1.849^*$	$6.079 \pm 0.265^*$	$4.362 \pm 0.255^*$

model and the actual marker positions obtained from the motion capture. Inverse dynamics was conducted using the joint angles resulting from inverse kinematics and GRF data to compute the net joint moment. Because stance phase is responsible for providing support, stability, and propulsion, and any mismatch in joint kinematics and mechanics during this phase can have a significant impact on overall gait performance, we separated the results for both the full gait cycle and the stance phase alone.

E. Data Analysis

Ankle and knee joint angles were computed for both intact and prosthetic sides using the inverse kinematics function in OpenSim with two models: one with and one without prosthesis. Similarly, ankle and knee joint moments were computed using the inverse dynamics function in OpenSim. In addition, angles and moments for both the ankle and knee joints of the prosthesis were recorded from the encoders and current information, respectively. Considering the joint angles and moments from the prosthetic sensors as the ground truths, root-mean-square errors (RMSEs) were computed for both joint angles and moments which were estimated from the two OpenSim models.

Given the study's design incorporating only a single subject, it is inappropriate to conclusively attribute the observed differences in the accuracy of inverse kinematics and dynamics to the inclusion of the prosthesis in the OpenSim model using statistical analysis. However, to examine the feasibility of inclusion of the model of prosthesis, we executed t-tests on all Root Mean Square Error (RMSE) values. As the paired t-test is a parametric test that presumes the data conforms to a normal distribution, we initially applied the Shapiro-Wilk test to verify normality. The p-value we obtained was greater than 0.05, indicating that the RMSE data strongly adhered to a normal distribution. However, an exception was noted in the case of the ankle angle in the full phase data for the model incorporating a prosthesis, where normality could not be assumed. This discrepancy prompted an adjustment in our statistical strategy, leading to the replacement of the parametric paired t-test with the Wilcoxon signed-rank test, a non-parametric equivalent. The application of this alternative test catered to the non-normal distribution exhibited by the ankle angle data in the prosthesis model, thereby ensuring a more dependable and suitable statistical evaluation. For our t-tests, we set the level of significance to $\alpha = 0.01$. All statistical analyses were executed using Python, supplemented

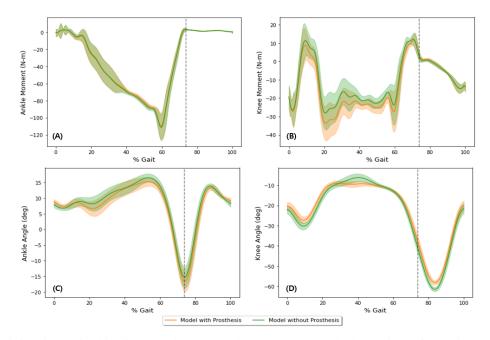


Fig. 4: Intact side joint kinematics/kinetics. Results presented as mean \pm SD (shaded region) of 13 gaits. Orange corresponds to the model with the prosthesis, and green denotes the model without the prosthesis. The black dashed line separates the stance phase from the swing phase.

by the SciPy library (version 1.10.1) [27].

III. RESULTS

Fig. 3 shows inverse kinematics and inverse dynamics results and actuator data (ground truth) at the ankle and knee joints of the prosthesis side of the model with and without prosthesis characteristics. According to Fig. 3(A) and (B), the dynamics results of the model with prosthesis are closer to the actuator data, and in the swing phase, both models show completely different results from the encoder data. According to Fig. 3(C) and (D), the model with prosthesis is closer before reaching the peak value of the swing phase but not after reaching the peak value. In Table II, we report RMSE values for both the full gait cycle and the stance phase alone. Table II shows that all components of RMSE of the model with prosthesis are smaller than the model without prosthesis, except for ankle angle at the full phase. During the stance phase, statistically significant differences were observed for all variables except ankle torque. However, in the full gait cycle, statistically significant differences were found only for the ankle angle. Fig. 4 shows the results of inverse kinematics and inverse dynamics of two models on the intact side. Except for the knee moment during stance phase, similar results are shown on the intact side for the two models.

IV. DISCUSSION

This study aimed to compare the performance of two models in simulating the kinematics and dynamics of the ankle and knee joints of the prosthetic side of a lower-limb model: one model incorporating the prosthesis characteristics and the other without these characteristics. The results demonstrate

the importance of including the prosthesis characteristics in the model to accurately simulate the joint behavior. From Fig. 3(A) and (B), it is evident that the model with prosthesis offers a closer match to the actuator data than the model without prosthesis except during swing phase. This is also supported by the RMSE of ankle and knee torque during stance phase in Table II. This could suggest that the model with prosthesis better reflects the dynamics features of a real powered prosthesis during stance phase. However, the swing phase dynamics reveal considerable differences between the two models and the actuator data. The marked differences during swing phase could be attributed to the distinct approaches for generating trajectories when contact is not present. The actuator torques are forced to follow the desired trajectories, resulting in a high torque value. On the other hand, the torques obtained from inverse dynamics are based on inverse kinematics solutions without considering factors such as stiffness, friction, etc. Consequently, the computed torques tend to be small in magnitude.

Although the model with prosthesis characteristics demonstrates better performance before reaching the peak value of the swing phase, as shown in Fig. 3(C) and (D), it does not maintain the same level of accuracy after reaching the peak value. However, this discrepancy may simply be a shift due to differences between the model and the actual prosthesis. Since the shift did not affect the dynamics of the results, as shown in Fig. 3(A) and (B), the angular velocity and acceleration can be considered well-reflected. Rather, as shown in Fig. 3(B), the model incorporating a prosthesis more accurately captures a negative moment in the knee during the swing phase which is crucial for facilitating

smooth and energy-efficient limb movement in prosthesis side gait as well as intact side gait during the swing phase [28], [29].

In the t-test analysis, it was observed that statistically significant differences were present for all variables except for ankle angle for the full gait cycle. In conjunction with the RMSE results, these findings may indicate that the model with prosthesis provides a more accurate representation of the kinematic and kinetic features of prosthesis side gait.

Our analysis revealed that inverse kinematics and inverse dynamics results on the intact side showed a similar trend to a normal gait data found in [30]. However, some differences with the normal gait data were observed, which can be attributed to the compensatory gait patterns adopted by the subject due to the presence of the prosthesis on the other side. It is well-established that individuals with lower-limb prostheses often exhibit compensatory mechanisms to maintain balance and stability, which can result in deviations from the typical gait patterns observed in able-bodied individuals [31], [32].

Some limitations of this study should be acknowledged. First, the prosthesis model used may not capture all the intricacies of real-world prosthetic devices, which could affect the accuracy of the results. Second, the experiment was conducted on only one subject. Therefore, it would be beneficial to perform similar analyses with a larger and more diverse population to validate the findings and generalize the conclusions.

V. CONCLUSIONS

This study demonstrates the importance of including prosthesis characteristics when modeling the kinematics and dynamics of lower-limb prostheses. The model with prosthesis characteristics outperforms the model without these characteristics in terms of accuracy and consistency with the encoder data. These findings have implications for the development of more accurate computational models for prosthetic limb users, which can ultimately lead to improved prosthetic designs, better rehabilitation strategies, and enhanced quality of life for amputees.

Building upon the findings of this study, several avenues for future research can be explored to further improve the understanding and application of computational models with prosthesis. First, To strengthen the generalizability of the results, future studies will involve a larger and more diverse group of participants, including individuals with varying levels of amputation. Second, we will also consider the influence of external factors, such as varying walking surfaces, slopes, and environmental conditions, on prosthesis performance. Investigating the effects of these factors on the accuracy of computational models will help refine and improve the models, making them more robust and adaptable to real-world situations.

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