

SIGNIFICANCE

- This could aid in the creation of bipedal devices, such as prostheses, with more efficient and consistent trajectory generation.
- This study can be added to those in other fields in order to create more general best practices of direct collocation problem setup.

INTRODUCTION

- Applications of bipedal robotics, such as rough terrain traversing robots [1] and prostheses [2], may require reference trajectories.
- One way to solve such optimization problems is with direct collocation [3].
- Different aspects of problem setup and implementation, such as collocation [4] and differentiation [5] methods, may affect the efficiency of solution and accuracy of the result.
- The aim of this study [6] was to compare how such factors affect bipedal walking gait generation.

WALKING MODELS AND NONLINEAR PROGRAMS

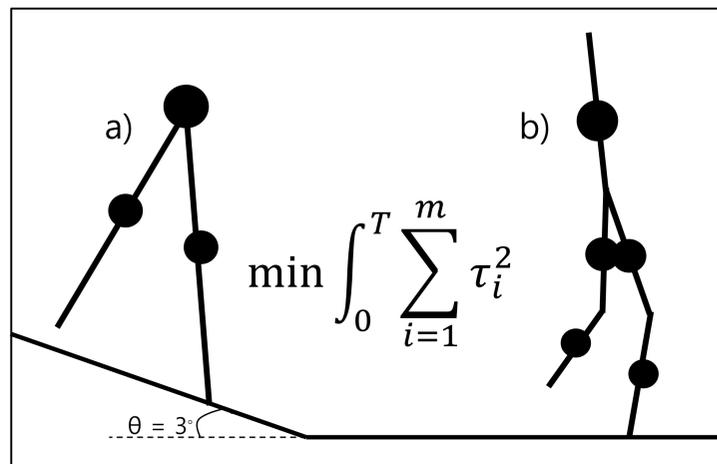


Fig. 1 Two bipedal walkers used in this study, a) compass walker and b) kneed biped. In both cases, control effort was minimized.

Walking models

- 2-DOF, 1-DOA compass walker
- 5-DOF, 4-DOA kneed walker with torso

Decision variables

- Positions, velocities, accelerations (one test case)
- Input torques
- Time step

Constraints

- Kinematic (step length, initial foot positions, periodicity, foot clearance, knee hyperextension)
- Dynamic (continuous, discrete)

DIRECT COLLOCATION

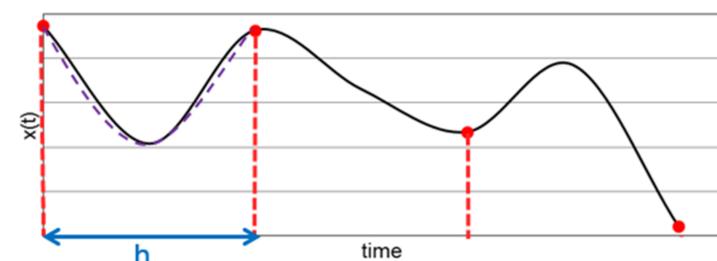


Fig. 2 Illustration of direct collocation [6]. The collocation points are in red, the time step is in blue, the spline is in purple, and the unknown true trajectory is in black.

- Discretize continuous-time trajectory at collocation points and solve for decision variables at those points in time
- Use spline interpolation to get approximated continuous result
- Trapezoidal (TPZD) – linear dynamics/quadratic states
- Hermite-Simpson (H-S) – quadratic dynamics/cubic states

SYMBOLIC AND NUMERICAL DIFFERENTIATION

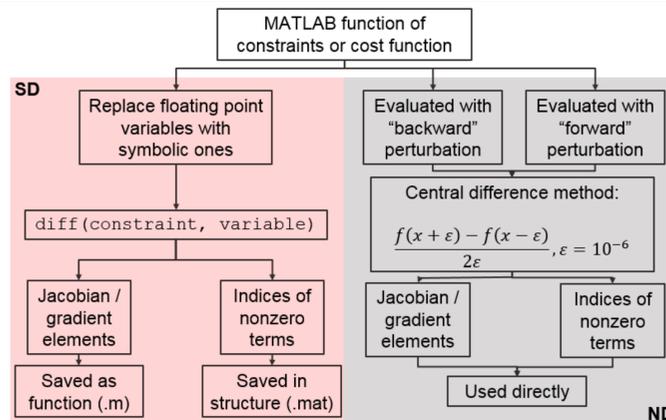


Fig. 3 Descriptions of differentiation implementations tested. SD and ND indicate symbolic and numerical differentiations, respectively.

SIMULATION SETUP

Testing

- MATLAB R2019a with mexIPOPT [7] NLP solver
- 100 runs/test case for compass walker
- 20 runs/test case/number of points for five-link biped
- Randomized initial guess
- Test cases provided in Table 1

Table 1 Test cases.

Case	Compass		5-Link	
	Collocation Method	Jacobian Differentiation Method	Acceleration Computation	Jacobian Differentiation Method
1	H-S	ND	Symbolic	SD
2	TPZD	ND	Symbolic	ND
3	H-S	SD	Numerical	ND
4	TPZD	SD	In DVs	ND

Analysis

- Runtime and runtime per iteration
- Deviation from baseline gait
- Analyzed with ANOVA

RESULTS

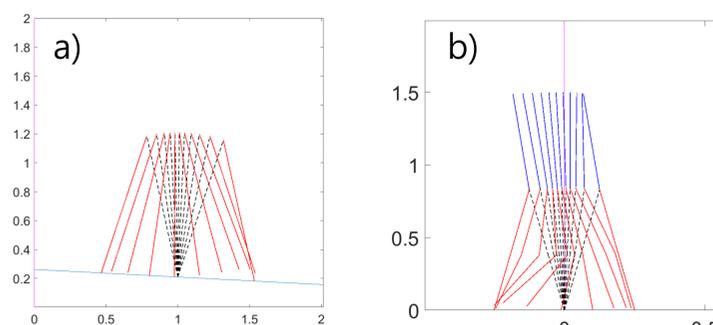


Fig. 4 Baseline gaits generated for a) compass and b) five-link bipeds. Most runs converged to these gaits [6].

Compass walker

- ND was shown to be faster than SD.
- H-S was slower than TPZD, but it also deviated less from the baseline gait.

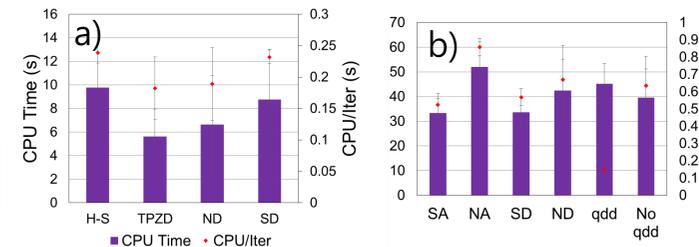


Fig. 5 Time-related data for a) compass and b) five-link bipeds [6]. The axis titles in b) are the same as those in a).

Five-link biped

- Runs where joint accelerations were evaluated symbolically were faster than those where they were evaluated numerically.
- Runs with acceleration in the decision variables were faster per iteration but required far more iterations.
- Symbolic differentiation quickly becomes more complex with increased degrees of freedom.
- All runs with acceleration in the decision variables converged to the expected gait. Controlling for alternative gaits, these runs were less accurate.

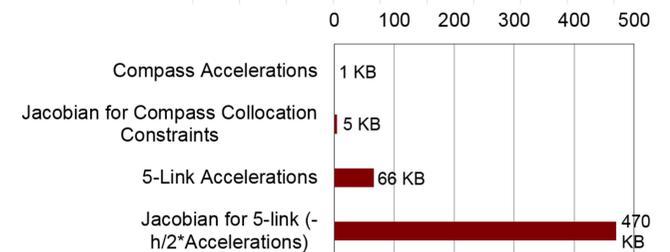


Fig. 6 Comparison of file sizes for compass and five-link bipeds [6].

CONCLUSION

Recommendations

- For the compass walker, ND and TPZD should be used.
- For the five-link biped, accelerations should be included in the decision variables.
- If the accelerations are not included, symbolic acceleration and numerical Jacobian calculation should be implemented.

Other insights

- Tradeoffs can be necessary between factors such as accuracy and efficiency.
- Different combinations of settings may be beneficial in specific situations.

FUTURE WORK

- Different collocation and differentiation methods (e.g., automatic differentiation) can also be tested and compared.
- The efficiency of implementation in different programming languages can be tested.
- More runs and different combinations of settings can be tested.

References

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