Invariant Density Analysis: Modeling and Analysis of the Postural Control System Using Markov Chains

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***Abstract*—In this paper, a novel analysis technique, invariant density analysis (IDA), is introduced. IDA quantifies steady-state behavior of the postural control system using center of pressure (COP) data collected during quiet standing. IDA relies on the analysis of a reduced-order finite Markov model to characterize stochastic behavior observed during postural sway. Five IDA parameters characterize the model and offer physiological insight into the long-term dynamical behavior of the postural control system. Two studies were performed to demonstrate the efficacy of IDA. Study 1 showed that multiple short trials can be concatenated to create a dataset suitable for IDA. Study 2 demonstrated that IDA was effective at distinguishing age-related differences in postural control behavior between young, middle-aged, and older adults. These results suggest that the postural control system of young adults converges more quickly to their steady-state behavior while maintaining COP nearer an overall centroid than either the middle-aged or older adults. Additionally, larger entropy values for older adults indicate that their COP follows a more stochastic path, while smaller entropy values for young adults indicate a more deterministic path. These results illustrate the potential of IDA as a quantitative tool for the assessment of the quiet-standing postural control system.**

***Index Terms*—Balance, Center of Pressure (COP), Nonlinear Biodynamics, Postural Control, Stochastic Mechanics.**

I. Introduction

POSTUROGRAPHIC data collected during quiet stance using force platforms are widely used to assess human postural control. In particular, force plate data are commonly used to estimate the subject's quiet standing center of pressure (COP) especially in laboratory settings. COP measures have been used to investigate human postural control, sensorimotor degradation due to aging, and balance disorders [1]–[7]. Traditionally, COP data have been analyzed using measures that describe shape, speed, or frequency content of the trajectory, such as standard deviation, mean velocity, mean distance, total excursion length, range, maximum distance, peak frequency, or mean frequency [8]–[13]. Unfortunately, these parameters do not provide insight into the physiological system as a whole and have been shown to have questionable reliability [9], [10], [14].

Stochastic models of the COP trajectory have been used to more fully describe the quiet-standing postural control system. These models tend to approximate the behavior of the COP as a nearly random walk. A random walk in this case is a mathematical model where, at each step, the trajectory jumps to another site according to some probability distribution. Collins and De Luca [15] modeled COP data as a nearly random walk, and then used stochastic analysis techniques to quantify underlying deterministic behavior in the data. In their work, stabilogram diffusion analysis (SDA) was used to identify regions of short-term (open-loop) and long-term (closed-loop) postural control strategies during quiet standing. While SDA characterizes time-dependent behavior of the COP trajectory, this technique does not capture the positional dependence of the data. Furthermore, SDA can only provide summary information about the human postural control system; it cannot provide specific information about or recreate the human postural control system or recreate the actual sway behavior [16]. In addition, Ornstein–Uhlenbeck processes have been used to model COP data [16], [17]. This process models the apparent random walk of the COP trajectory as Brownian motion and compares the current location to the long-term mean of the converged trajectory. However, Ornstein–Uhlenbeck processes do not fully capture the variance of the random walk [16]. Additionally, a 2-D Langevin equation has been used to model COP data as a random walk [18]. The Langevin equation models Brownian motion in potential fields to formulate the equations of motion for the COP trajectory [19]. Bosek *et al*. [18] used a 2-D Langevin equation to approximate the short-term region of the stabilogram diffusion plot. While these latter models [16]–[18] can detect deterministic behavior in the stochastic random walk of the COP, they provide only a single control mechanism or governing equation for the system. Furthermore, since the models were constructed using a fit to the variance function of the diffusion process in the random walk, they do not provide evolutionary properties of the time series data [16]. In contrast, Rasku *et al*. applied a hidden Markov model to COP data during quiet standing [20]. Transition matrices were constructed for both healthy young adults and an older patient group with Meniere's disease. The models captured the evolutionary properties of the data and were used to differentiate the two groups with a resulting sensitivity range of 70–90%. However, Rasku *et al*. did not use the models to provide biomechanical or physiological insight of the subject's behavior.

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In this paper, an alternative approach for the analysis of a reduced-order model of the quiet-standing postural control system is considered, invariant density analysis (IDA). This approach uses a reduced-order Markov chain model of the COP trajectory, in place of closed-form equations, to describe the evolution of the state [21]. IDA is used to quantify the dynamics of the system itself and not just provide statistical description of system behavior as with traditional COP measures (e.g., [8]). This approach takes into account system evolution in terms of time (e.g., the evolution of the probability distribution into the invariant density distribution) and space (e.g., state-dependent transition probability). The remainder of this paper proceeds as follows. Section II presents the details of the Markov chain model and IDA. Section III describes the experimental methods used to develop and demonstrate IDA. Section IV considers the implications of these results in the context of human postural control. Section V provides concluding remarks.

II. Mathematical Method

The postural control system is a complicated dynamical system. It is generally not possible to derive simple closed-form system models. We, therefore, propose a data-driven approach to construct a reduced-order Markov-chain model from COP data to characterize the long-term behavior of the quiet-standing postural control system. The COP was treated as an output of the dynamical system that results from the stabilizing mechanisms of the human postural control system. This approach has its roots in discretization of dynamical systems using set-oriented methods [21]. Here, we present background on system modeling, methods for construction of a discrete Markov chain model from COP data, the calculation of the invariant density, and the introduction of IDA to characterize the postural control system during quiet stance.

A. System Modeling

Dynamical systems are approximated using mathematical models to describe the states of the system and evolution of those states. The evolution of the system can be a deterministic or stochastic process. Deterministic models have only one possible future state that evolves from the current state (e.g., differential equations that describe the motion of a pendulum). Stochastic models have several potential states, and the likelihood that the stochastic system evolves to a particular state can be described using a probability distribution. A stochastic process is considered to be a "Markov chain" if future states are independent of all past states and therefore only relies on the present state [22]. That is, X is a Markov chain if

$$\begin{aligned}&\mathbf{P}(X_{n+1}=x_{n+1}|X_n=x_n,\ldots,X_0=x_0)\\&=\mathbf{P}(X_{n+1}=x_{n+1}|X_n=x_n)\end{aligned}\tag{1}$$

for a stochastic process $X = (X_1, X_2,...)$ with state space X and probability measure $\mathbf{P}$. A one-step evolution of the state is called a transition, and the probabilities associated with possible state transitions are called transition probabilities. Assuming that there is a finite set of states, the transition probabilities can be expressed in a transition matrix, P. The transition probabilities in P govern the evolution of the Markov chain, and the probability distribution evolves as

$$\lambda_{n+1}=\lambda_n\,P\tag{2}$$

where λ_n is the distribution of the state at the nth iteration. If the Markov chain is irreducible and recurrent [22], λ_n converges to a unique steady-state distribution π, which is also equivalent to the left eigenvector of P with eigenvalue 1:

$$\pi=\pi\,P\tag{3}$$

where π is referred to as the *invariant density*. π is important because it does not depend on the initial system distribution and defines the long-term system behavior.

A Markovian model based on COP data is able to capture both the steady-state and time-evolving properties of the postural control system. In this model, steady-state behavior is represented by the invariant density and time-evolving behavior is represented by the transition matrix. Even though the invariant density may be computed directly from time series COP data, a discrete Markov chain model was used here because the Markovian framework provides additional information about the dynamical behavior of the system (e.g., rate of convergence (second eigenvalue of P) and the entropy of the system).

B. IDA Analysis

1) Markov Chain Model Construction From Data: In this study, a discrete Markov chain model was used to extract dynamical information from experimental COP data. For each COP dataset, the Markov model and invariant density were constructed in the following manner. First, the COP data were zero-mean adjusted to the centroid of the data. The state space was partitioned and discretized by concentric circles emanating from the centroid with radii increasing from 0.0 mm by steps of 0.2 mm. The width of the rings was determined by the noise level measured during a static weight calibration of the force platform used for COP collection. In other words, the width of the ring was the same as the standard deviation of the COP data (rounded to the nearest tenth of a millimeter) during the calibration. Second, the transition matrix P was constructed by computing transition probabilities for all states (4). Fig. 1(a) is a simplified illustration of the finite state space used to construct the transition matrix for the model. In this example, the state space was discretized into four states (rings 1–4). The 4×4 transition matrix P that describes the state transitions of the COP for this example is given in Fig. 1(b). For the work presented here, the dimension of the state space was between 100 and 200 states. Third, the invariant density π was computed by solving for the left eigenvector of P (3), with an eigenvalue of 1; thus, π describes the probability of finding the COP in a given state.

$$P_{ij}=\frac{\text{number of transitions from state } i \text{ to state } j}{\text{total number of transitions within or out of state } i}.\tag{4}$$

2) Parameterization: Even though the invariant density π describes COP behavior, a parameterization of π facilitates quantification of the data and offers enhanced understanding

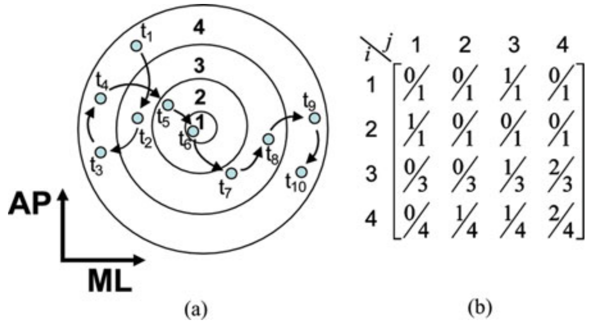

Fig. 1. (a) Illustration of the states (concentric circles) used to define the location of the COP. The blue dots represent an example COP trajectory made up of ten data points (t_1 to t_{10}). The elements of the probability transition matrix *P* are calculated directly from the COP data. (b) Transition matrix *P* for the given trajectory. Refer to (4) to obtain *P*.

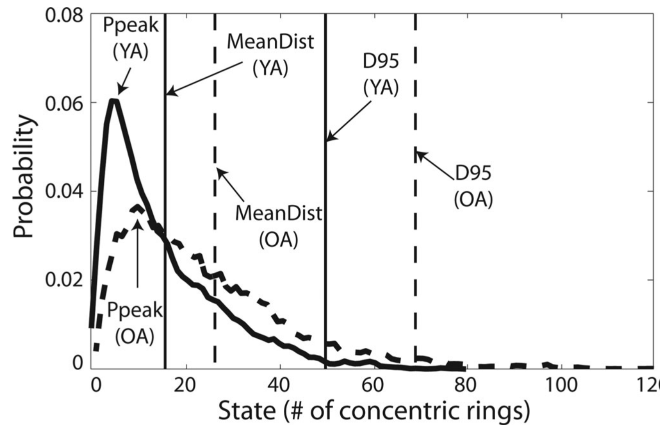

Fig. 2. Example plot of the invariant densities and IDA parameters of both young (YA, solid) and old (OA, dashed) adult subjects showing the probability of the location of their COP.

of subject behavior. Five parameters were used to characterize the discrete Markov chain model.

1) *Ppeak* (*Largest probability of the invariant density*): A larger *Ppeak* value indicates a higher probability that the COP will be driven to a particular state. *Ppeak* is unitless.
2) *MeanDist* $\sum_{i\in I} i\pi(i)$ (*Weighted average state (or average location) of the COP, where I is the set of all possible states*): *MeanDist* is a measure of the average distance that the COP moves away from the centroid. Larger values signify greater average travel of the COP. *MeanDist* has units of rings, or mm after multiplying the number of rings by ring width.
3) *D95* (*Largest state at which there is a 95% probability of containing the COP*). This parameter provides insight into how far the COP diffuses from the centroid. *D95* has the same unit as *MeanDist*.
4) *EV2 (Second largest eigenvalue of the transition matrix)*: This corresponds to the rate of convergence to the invariant density. *EV2* describes how quickly the COP will reach its invariant distribution and how sensitive the process is to perturbation [23]. A smaller *EV2* indicates a lower sensitivity. *EV2* is unitless.
5) *Entropy* $(-\sum_{i\in I} \pi(i)\log_2 \pi(i))$ (*Measure of randomness or uncertainty of the system*): Low entropy corresponds to a more deterministic system and high entropy refers to a more stochastic system. This parameter is equivalent to the concept of Shannon entropy [24]. *Entropy* has units of bits.

Fig. 2 shows a plot of two invariant densities and associated IDA parameters (*Ppeak*, *MeanDist*, and *D95*) that can be identified on the invariant density plot.

III. Experimental Studies

Two experimental studies were used to demonstrate the efficacy of the IDA approach. Study 1 was conducted to determine if data from multiple short trials could be combined to create a dataset of sufficient length for IDA. Since IDA examines long-term quiet-standing behavior, it requires COP data on the order of minutes. Combining multiple short trials into a single long trial was of interest because COP data are commonly collected from multiple trials in durations on the order of seconds. We examined whether or not the invariant densities based on ten 30-s trials or a single 5-min trial were statistically different. A secondary outcome of this study was the identification of the minimum time required to reliably compute the invariant density.

Study 2 examined whether IDA parameters can explain age-related changes seen in postural control behavior. Quiet-standing trials were conducted by adult subjects from three age groups: young, middle-aged, and old. Age-related changes to the postural control system, as assessed through previous measures of COP, have resulted in greater postural sway [25]–[28].

A. Experimental Setup and Procedures

For both studies, subjects had no balance issues and no history of significant trauma to the lower extremities or joints. All procedures were approved by the university Institutional Review Board, and all participants gave informed consent. For all trials, the subjects were instructed to stand quietly on a force platform (AMTI, model BP600900; Watertown, MA). Each subject self-selected a comfortable stance on the platform and stood with arms crossed at the chest while looking at a picture placed at eye level 3 m in front. Foot tracings were made to ensure consistent foot placement because the platform was rezeroed between each trial. Data were sampled at 1000 Hz. It is likely that a reduced sampling rate of 100 Hz may be used in the calculation of the invariant density without loss of model fidelity because of its commonality in the literature [8], [15], [16]. Force platform data were not filtered because the discretized state space takes into account noise present in the data.

1) Study 1—IDA Validation: Each subject performed the ten 30-s trials followed by the 5-min trial (see the next section for more details). The ten 30-s trials were combined into a single 5-min trial using the following approach. COP data were zero-mean adjusted about the data centroid. Then, the ten trials were concatenated with each other. Because we were interested in the distribution of the points in the predetermined states and not in the continuity of the COP trajectory, discontinuities between the ten 30-s trials did not affect the analysis.

Next, to investigate the time needed for a subject's COP data to reach its invariant density the 5-min trial was broken into ten intervals of increasing length, such that the 30-s trial was calculated using the first 30 s, the 60-s trial used the first 60 s, etc. IDA analysis was applied to each interval. The duration of time required for the error norm to fall within 5% of the value calculated from the 5-min (300 s) steady-state data was identified as sufficiently long to compute the invariant density. The error norm was defined as follows:

$$E_j = \sqrt{\sum_{i=1}^{5}\left(\frac{Param_{i,j} - Param_{i,300}}{\max\limits_{k}(Param_{i,k}) - \min\limits_{k}(Param_{i,k})}\right)^2} \times 100\%$$
$$(j, k = 30, 60, 90, ..., 300) \quad (5)$$

where $Param_{i,j}$ is the ith parameter value for j seconds ($i =$ *Ppeak*, *MeanDist*, *D95*, *EV2*, *Entropy*). Normalized values are used in (4).

2) Study 2—IDA Analysis of the Effect of Age on Quiet Stance: Data from a previous study [29] of 45 subjects were used for study 2 (see the next section for more details). In this previous study, ten 30-s trials were collected from each subject. The methodology used in study 1 to concatenate the data was also used here. The concatenated data were then used to construct the discrete Markov chain models of the COP, as described in Section II-B1. Once the models were derived from the data subject-specific, IDA parameters were computed using the methodology described in Section II-B2.

Paired t-tests were used to examine differences between IDA parameters determined from one 5-min trial and ten 30-s trials in Study 1. One-way analysis of variance (ANOVA) was used to test the effect of age on the IDA parameters in Study 2 (SPSS Inc., Chicago, IL; v17). Tukey's Honestly Significant Differences (HSD) tests were used for *post hoc* comparisons in Study 2. Level of significance was $\alpha = 0.05$.

B. Subject Demographics

1) Study 1—IDA Validation: Ten young adult subjects were recruited for Study 1. Five male subjects of mean (standard deviation) height 182.3 (4.6) cm, weight 77.6 (4.8) kg, and age 22.2 (3.83) years and five female subjects of mean height 159.0 (4.5) cm, weight 61.0 (5.5) kg, and age 21.2 (1.79) years participated in Study 1.

2) Study 2—IDA Analysis of the Effect of Age on Quiet Stance: Subjects were divided into three groups of 15: young (YA, age: 19–30 years, height 168.8 (13.0) cm, weight 67.0 (9.5) kg), middle-aged (MA, age: 42–53 years, height 171.3 (9.5) cm, weight 76.3 (14.8) kg), and old adults (OA, age: 62–80 years, height 164 (1) cm, weight 76.9 (17.1) kg).

C. Results

1) Study 1—IDA Validation: The invariant densities and IDA parameters from the concatenated 30-s trials and the single 5-min trial compared well. Paired t-tests found no significant differences between the concatenated and the continuous time trials (see Table I). *Post hoc* analyses found high power for all parameters (>92%) suggesting that the concatenated data can be used for the calculation of IDA parameters. Additionally, it was found that the error norm reached the 5% threshold after 210 s (see Fig. 3).

TABLE I
COMPARISON OF IDA PARAMETERS FOR TEN 30-S TRIALS AND ONE 5-MIN TRIAL (MEAN ± SD)

	Ten 30 s trials	*One 5 min trial*	p^*
Ppeak	0.042 ± 0.012	0.038 ± 0.008	0.37
MeanDist	4.31 ± 1.22	4.68 ± 0.97	0.23
D95	10.17 ± 3.29	10.24 ± 2.04	0.93
EV2	0.998 ± 0.001	0.998 ± 0.002	0.39
Entropy	5.58 ± 0.41	5.66 ± 0.27	0.49

* p-value from univariate ANOVA examining effect of concatenating ten 30 s trials

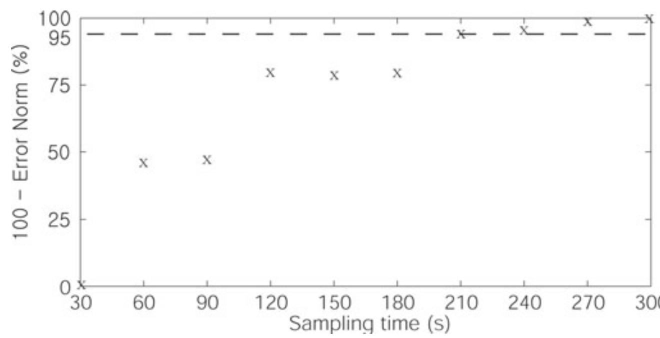

Fig. 3. Error norm between IDA parameters calculated from 300 s of data and shorter time periods. Error norm was normalized such that error nom was 100% at 30-s data and 0% at 300-s data.

2) Study 2—IDA Analysis of the Effect of Age on Quiet Stance: Significant age-related differences for all five IDA parameters were found (see Table II). *Post hoc* tests revealed statistically significant differences between YA and MA and between YA and OA for all IDA parameters, and between MA and OA for all parameters except *EV2*. *Ppeak* was found to be smaller with older ages. *Entropy*, *MeanDist*, and *D95* were greater with older ages. *EV2* was smaller for YA than for MA. However, *EV2* was not significantly different between MA and OA.

IV. DISCUSSION

In this paper, we have outlined the procedure for constructing and characterizing a reduced-order finite state Markov chain model of the quiet-standing postural control system. IDA parameters were developed to quantify information about the long-term behavior of the system captured by the model. Additionally, we presented two studies illustrating the practicality and benefits of this approach.

In Study 1, we verified that ten 30-s quiet-standing trials can be combined to form a dataset suitable for IDA. We found no statistical difference and high statistical power between IDA parameters calculated from one 5-min or ten concatenated 30-s trials (see Table I). Even though the initial COP position may

TABLE II
IDA PARAMETERS FOR EACH AGE GROUP

	Young *n=15*	*Middle* *n=15*	*Old* *n=15*	p^*
Ppeak	0.048 ± 0.010†	0.039 ± 0.007‡	0.031 ± 0.004!!	< 0.001
MeanDist	3.56 ± 0.68†	4.24 ± 0.68‡	5.44 ± 0.86!!	< 0.001
D95	7.98 ± 1.61†	9.67 ± 1.68‡	11.66 ± 1.47!!	< 0.001
EV2	0.997 ± 0.002†	0.999 ± 0.001‡	0.999 ± 0.001	< 0.001
Entropy	5.31 ± 0.30†	5.58 ± 0.24‡	5.88 ± 0.18!!	< 0.001

* *p*-value from ANOVA examining effect of age
† Young and old adults are significantly different
‡ Young and middle-aged adults are significantly different
!! Middle-aged and old adults are significantly different

vary for each trial, invariant density is always the same [22]. Therefore, multiple short trials can be used to calculate IDA parameters to prevent subject fatigue or boredom during testing. Furthermore, Study 1 determined that 210 s worth of COP data were the minimum time required for reliable computation of IDA parameters for young adults (see Fig. 3). However, further work is needed to validate the feasibility of concatenating data from test groups other than young healthy adults.

In Study 2, as it is anticipated that the concatenation of data in Study 1 would hold true for other populations, IDA showed significant differences between data from YA, MA, and OA (see Table II). For the YA, *Ppeak* was significantly larger, while both *MeanDist* and *D*95 were significantly smaller than the older population. Larger *Ppeak* and smaller *MeanDist* values result from invariant densities with noticeable peaks in the probability distributions located close to the centroid (see Fig. 2). In contrast, the OA group had smaller peaks and more uniform distributions. Additionally, larger *MeanDist* and smaller *Ppeak* values in OA illustrate that the COP wanders further from the centroid and was less likely to be found in any particular state. The larger *Entropy* value for OA indicates that the COP follows a more stochastic path, while a smaller *Entropy* value for YA indicates more deterministic information in the data. This can be interpreted as YA using a greater degree of "active control" to keep the COP close to the centroid compared to OA. The reduction in "active control" seen in the OA data could be due to the reduced muscle strength of the older population [29], [30]. Finally, the second eigenvalue *EV2* was significantly smaller for YA indicating that their COP data converge more quickly to steady-state behavior. This result suggests that younger subjects would be more robust to perturbation than older subjects, in the sense that a mildly perturbed postural control system with smaller *EV2* can return to steady state faster than a system with a larger *EV2* value [23]. This interpretation of YA–OA results also applies to those of YA–MA and MA–OA. It is worth noting that *EV2* was the only parameter between the MA and OA groups that was not significant. This may suggest that even though the MA group sways less and in a less random manner than OA group, the robustness to perturbation between the MA and OA groups is similar.

In the literature, the 95% confidence circle area has frequently been used to quantify postural sway [8], [31], [32], [13]. *D95* is similar to the 95% confidence circle area in the sense that *D95* describes the distance to a concentric circle (or state) at which there is a 95% probability of containing the COP. However, they are different in that the 95% confidence circle area assumes that the data are normally distributed [8]. Therefore, *D95* can be used without any assumption of normal distribution since *D95* is directly computed from time series data.

While this study demonstrates the benefits of IDA, there are also limitations and areas for improvement that should be addressed in future work. In this study, IDA partitions the state space of the COP with concentric circles. However, this could be improved by introducing different partition shapes. For example, concentric ellipses instead of circles could be used to partition the state space since COP data fluctuate farther in the AP than ML direction. The lengths of major and minor axes could be determined from the standard deviations of COP data in both AP and ML directions. Additionally, the effect of foot placement during quiet standing on the IDA parameters should also be investigated. It has been demonstrated that postural sway parameters are affected by changes to foot width, base of support area, and foot opening angle [31]. Correlations or linear regression analysis between IDA parameters and foot placement could be conducted and used to create appropriate normalization procedures for the computation of more robust and reliable IDA parameters.

IDA parameters also have the potential to be affected by nonlinear force plate errors (e.g., bending of force platform due to loading) [33]. In this case, a recalibration of the force plates using a nonlinear method, such as [33], may enhance the accuracy of IDA parameters. IDA parameters, however, are not sensitive to relative location errors in the force plate data because the COP trajectory is zero-mean adjusted to the centroid of the data during computation of the invariant density.

IDA analysis can be affected by a low-resolution discretization of the state space. For example, if the distance between rings was doubled, *Ppeak* values would increase because the *Ppeak* state would now have a greater number of data points from the

COP trajectory. As such, it is advised to use the highest resolution possible for the state-space discretization. However, if the distance between states is smaller than the noise level of the sensor, then the noise itself could be falsely interpreted as COP movement between states.

Further, investigation of the second eigenvector of the transition matrix may have the potential to provide a more complete understanding of the embedded dynamics in the reduced-order model. Recently, the second eigenvector has been used to formulate an intuitive understanding of the dynamics for a finite state-space ergodic Markov chain by allowing the decomposition of the state space into essential features [34]–[36]. This potential decomposition of state space may provide additional insight into the human postural control system. For example, the IDA state space may be decomposed into two regions based on the sign of the second eigenvector. These two regions may have a connection with the concept of two-regime control (open loop and control loop) mechanism during quiet standing proposed by Collins and De Luca [15].

V. Conclusion

This paper introduced and demonstrated a new approach to characterize and provide greater insight into the long-term dynamical behavior of the postural control system, IDA. IDA successfully distinguished age-related differences in the dynamical behavior of the postural control system. Future applications of this technique have the potential to provide insight into changes seen in the quiet-standing postural control system of other populations.

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