

## RESEARCH HIGHLIGHT

- Our findings support the idea that human walking tends to have low angular momentum because it allows human gait to be more robust to disturbances.

## INTRODUCTION

### Trajectory Tracking

- The manner in which walking trajectories are obtained is critical, since a controller may not yield a stable walking gait if the trajectories which it tracks are not robust to disturbances.

### Human Walking

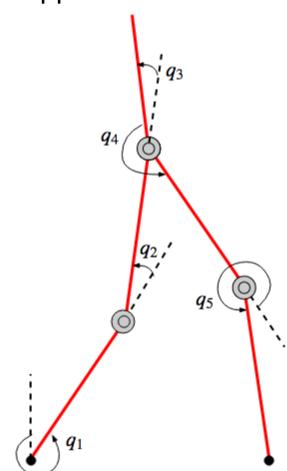
- Prior research has shown that angular momentum  $H$  about the whole body center of mass (COM) is highly regulated during human walking [1].

### Robustness of Trajectories

- The objective of this study is to examine the robustness to a disturbance of joint trajectories generated with different cost functions.
- Terms which facilitate the minimization of  $H$  are added to the cost function, and the performance of base-line trajectories is compared to the performance of trajectories which also minimize  $H$ .
- We would expect trajectories yielded by the cost function including  $H$  to be more robust.

## FIVE-LINK BIPED MODEL

- The simulated walking robot is the five-link biped shown in Figure 1(a). It has dimensions, masses, and inertias similar to that of a human for the shank and thigh, and uses the head-arms-torso (HAT) approximation for the upper body [2].



$$\min \sum \tau^2 + \alpha H_{spin}^2 + \beta H_{orb}^2$$

s.t.

- Dynamics = 0
- ImpactDynamics = 0
- JointRange  $\geq 0$
- TorqueLimit  $\geq 0$
- StepLength  $\geq 0$
- ImpactVelocity = 0

**Figure 1: (a)** Five link biped **(b)** Optimization formulation. Note that all inequality constraints are expressed with " $\geq 0$ "

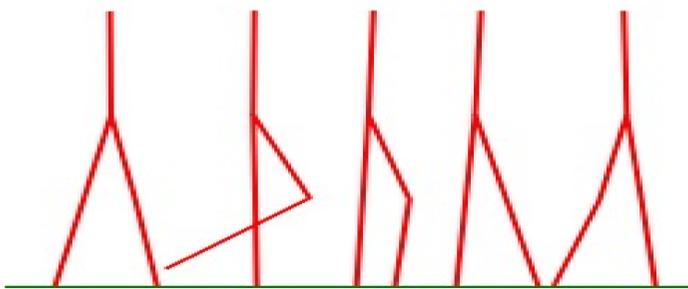
## METHODS

### Trajectory Generation

- The joint trajectories are generated via trajectory optimization using direct collocation, in which the desired optimal trajectories are discretized and used as the decision variables in the optimization [3].
- The accelerations, post-impact velocities, and impact forces are included with the decision variables in addition to those shown in [3].
- The optimization is done in Julia (v1.3) using JuMP as an interface and IPOPT as the solver.
- The Hermite-Simpson method is used for the direct collocation [3].
- The base-line cost function is the sum of the squared joint torques  $\tau^2$ . This cost function is augmented with the spin angular momentum squared  $H_{spin}^2$  (i.e.,  $\alpha = 1, \beta = 0$ ), orbital angular momentum squared  $H_{orb}^2$  (i.e.,  $\alpha = 0, \beta = 1$ ), or both  $H_{orb}^2$  and  $H_{spin}^2$  (i.e.,  $\alpha = 1, \beta = 1$ ).
- $H_{orb}$  is the angular momentum of a segment COM (point mass) in the reference frame of the whole-body COM.
- $H_{spin}$  is the angular momentum of a segment about its own COM (in its own reference frame).
- The optimization includes many constraints such as those for respecting the dynamics, a minimum step length of 0.2 m, periodicity constraints so that the step behavior is repeatable, limits on the joint angles, and impact at the heel strike.
- The details of these constraints can be found in [4].

### Forward Simulation

- A forward simulation is conducted in which a horizontal impulsive force is applied at 1.0 sec at the hip joint for a duration of approximately 0.4 sec, against the direction of the step.
- The biped is tasked with walking 10 steps without falling over.
- A PD controller is used for all simulations to track the trajectories, and center of mass forward progression is used as the phase variable (i.e., time parameterized by state variables).



## RESULTS

- The maximum force the biped withstands without falling over for a given cost function's set of optimal trajectories is reported in Table 1.
- To rule out the potential effects of step lengths on the maximum force, trajectories were generated for different fixed step lengths as well.

**Table 1:** Maximum force (N) of perturbation without falling for sets of trajectories with different cost functions and step lengths.

Step Length [m]	$\tau^2$	$\tau^2 + H_{spin}^2$	$\tau^2 + H_{orb}^2$	$\tau^2 + H_{spin}^2 + H_{orb}^2$
0.200	222	657	536	536
0.241	221	586	563	551
0.272	unstable	477	570	547

- Adding angular momentum into the cost function improved the maximum force tolerated in all cases.
- For  $H_{spin}^2$ , the maximum force decreased for increasing step length. The opposite trend is seen for  $H_{orb}^2$ .
- Surprisingly, the maximum force for the cost function with both terms was lower for the shorter two step lengths.

## CONCLUSIONS

- It may be advantageous to include one type of angular momentum over the other depending on the desired step length.
- The inclusion of angular momentum terms in the cost function increased the robustness of the biped to a disturbance without altering the controller.
- These results support the idea that human walking tends to have low angular momentum because it allows human gait to be more robust to disturbances.

## FUTURE WORKS

- This experiment could be expanded to include more types of perturbations and step lengths.
- The conclusions of this experiment should be verified with other controllers and maybe even other methods of trajectory generation.

## References

- [1] Popovic et al., ICRA, pp2405-2411, 2004.
- [2] Winter, Wiley, 4th ed., 2009.
- [3] Kelly, SIAM Review, 59(4), pp849-904, 2017.
- [4] Chao et al., IROS, pp1435-1440, 2019.