

# A Step Towards Generating Human-Like Walking Gait via Trajectory Optimization through Contact for a Bipedal Robot with One-Sided Springs on Toes

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09/27/17



# Outline

- Motivation and background
- Trajectory optimization through contact
- Modified framework
  - Hermite-Simpson method
  - Cost function for virtual flexible components
  - Complementary constraints for one-sided springs
- Result, conclusion and future work

# Motivation and background

Generating an efficient, robust walking gait with multiple contact domains is challenging

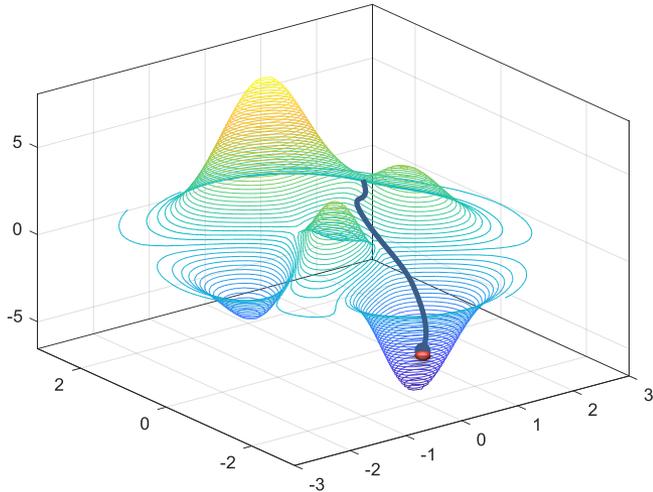
- The floating-based system is complex
  - High dimensions and nonlinearity
  - Non-convex constraints in kinematics and dynamics
- Existence of contact impact
- Combination of different contact domains
  - Different contact conditions (constraints) result in different dynamics
  - Mixture of full, over, or under actuation



# Motivation and background

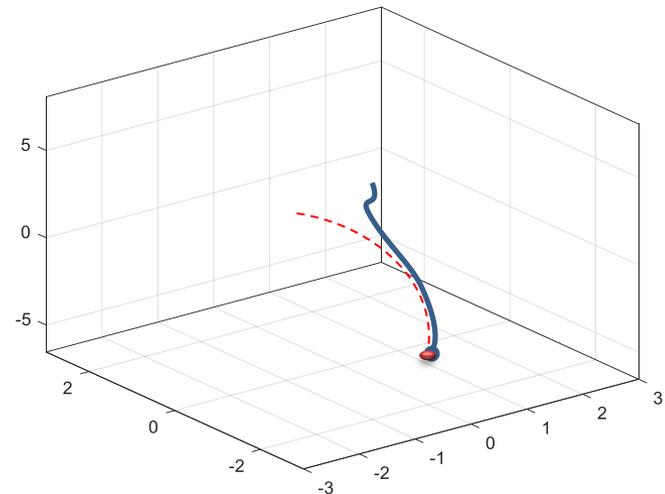
- Optimal control and trajectory optimization

closed-loop solution  $u = u(x)$



global optimal solution  
hard to compute  
lower dimensional problems

open-loop solution  $u = u(t)$



local method, suboptimal solution  
easier to compute  
higher dimensional problems

# Motivation and background

- Trajectory optimization is the collection of methods used to find the best choice of trajectory:  $x(t)$  and  $u(t)$ 
    - Single shooting
    - Multiple shooting
    - **Direct collocation**
- More accurate, harder to pose and solve
- Less accurate, easier to pose and solve

Continuous functions of time  $\longrightarrow$  Discrete set of real numbers

$$\begin{array}{ll} t \rightarrow [t_0, t_1, \dots, t_N] & t = \text{grid point number} \\ x(t) \rightarrow [x_0, x_1, \dots, x_N] & x_k = x(t_k) \\ u(t) \rightarrow [u_0, u_1, \dots, u_N] & u_k = u(t_k) \end{array}$$

# State of the arts: Trajectory optimization for walking motion generation with multiple contact domains

- Hybrid Zero Dynamics (HZD)-based trajectory optimization with direct collocation (Ames et al. 2015-16)
- Trajectory optimization through contact (Posa et al. 2012)

# State of the arts: Trajectory optimization for walking motion generation

**HZD-based**

**Optimization through contact**

Trajectory  
optimization  
approach

Direct collocation method

Optimization  
objective

Minimizing cost of transport (CoT)

$$CoT(\mathbf{x}) = \frac{\text{Energy}}{\text{Weight} * \text{Distance}} = \frac{1}{mgd} \sum_{k=1}^N \sum_i |u_{k,i} \dot{q}_{k,i}|$$

# State of the arts: Trajectory optimization for walking motion generation

	HZD-based	Optimization through contact
Impact handling	Hybrid-invariant constraints for phase-switchings	Time-stepping method
Predefined contact sequence	Required	Not required
Result sensitivity to initial guess	Relatively low	Relatively high
Transcription method for collocation	Hermite-Simpson method (cubic splines)	Trapezoid method (linear functions)

# Trajectory optimization through contact

$$\begin{array}{l}
 \text{minimize} \\
 x = \{h, x_0, \dots, x_N, u_1, \dots, u_N, \lambda_1, \dots, \lambda_N\} \\
 \downarrow \\
 \text{time step}
 \end{array}
 \quad
 \text{CoT}(x)$$

Complementary constraints for contact, e.g.

$$\begin{array}{l}
 G(x) \geq 0 \\
 H(x) \geq 0 \\
 G(x)^T H(x) = 0
 \end{array}
 \quad
 \begin{array}{l}
 toe_{height}(x) \geq 0 \\
 F_{toe} \geq 0 \\
 toe_{height}(x) F_{toe} = 0
 \end{array}$$

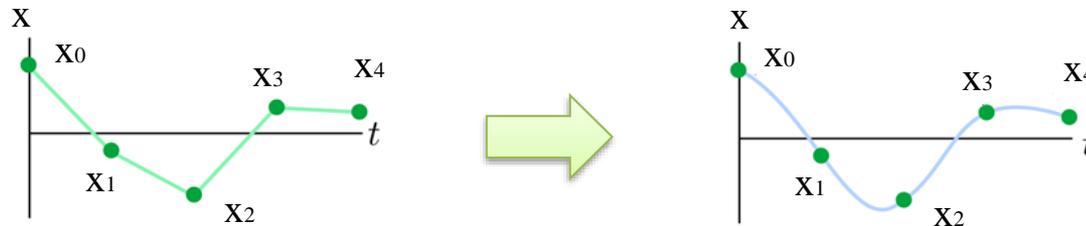
Kinematic, dynamic constraints, and periodic constraints

## Modified framework

- Hermite-Simpson method
- Cost function with virtual flexible components
- Complementary constraints for one-sided springs

# Modified framework (1/2)

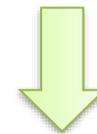
- Hermite-Simpson method



- Cost function with virtual flexible components

$$cost(\mathbf{x}) = \frac{1}{mgd} \sum_{k=1}^N \sum_i |u_{k,i} \dot{q}_{k,i}| + \omega \sum_{k=1}^N u_k^T u_k$$

$CoT(x)$

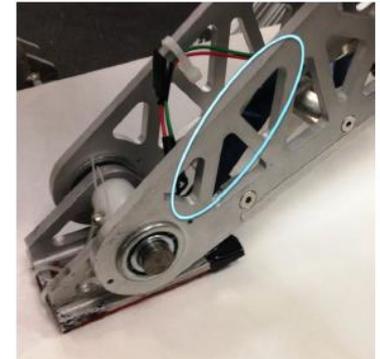


$$\omega \sum_{k=1}^N (u_k - kB_k q_k)^T (u_k - kB_k q_k)$$

## Modified framework (2/2)

- Complementary constraints for one-sided springs

$$\begin{aligned} G(x) &\geq 0 & k_{toe}\theta_{toe} &= (T_1 + T_2) - T^- \\ H(x) &\geq 0 & (T_1 + T_2)T^- &= 0 \\ G(x)^T H(x) &= 0 & T_1 T_2 &= 0 \\ & & \phi_{z,toe} T_1 &= 0 \\ & & \phi_{z,toe}, T_1, T_2, T^- &\geq 0 \end{aligned}$$



$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = Bu + J^T \lambda - J_{\theta,toe}^T T_1$$

# Software and Hardware

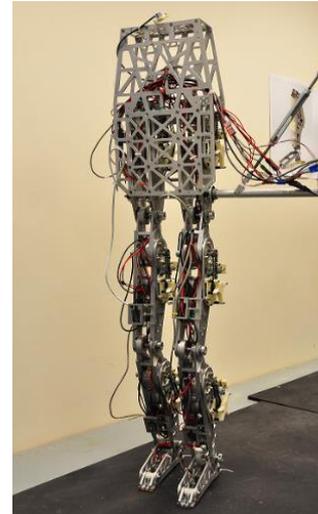
## Hardware (robot model)

### Planer bipedal robot Amber 3

- Height: 1.45 m
- Weight: 33.4 Kg
- 7-link, 6 DOFs

## Software

- MATLAB and mathematica
- Opt Solver
  - IPOpt
  - fmincon in MATLAB



# Optimization overview

## Opt. Problem Summary

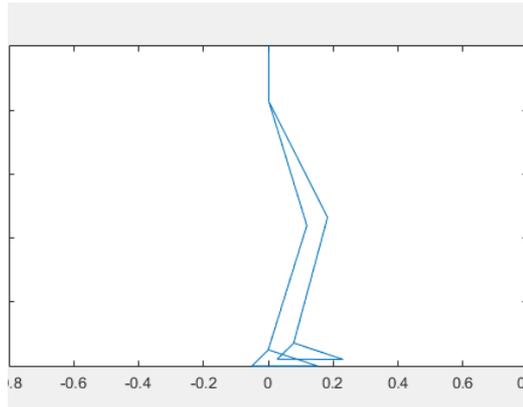
- Time duration: a half gait cycle
- 31 (discretized) collocation points
- 1334 free variables, 1709 constraints
- Can be solved in 1-6 minutes\*

Initial guess: ZMP-based flat-feet walking gait

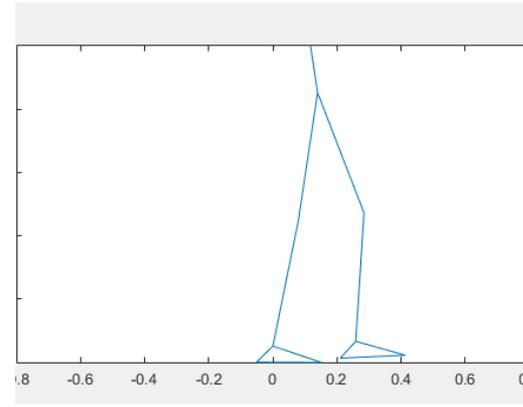
## Testing different set of contact constraints

- Sliding allowed contact constraints (SACC)
- Non-sliding contact constraints (NSCC)
- NSCC with one-sided springs (OSS)

# Simulation Result (1/2)



Initial guess

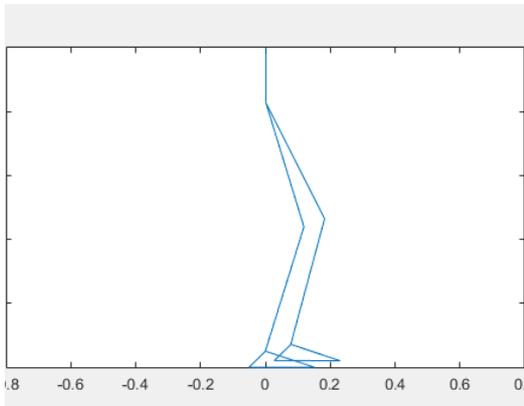


SACC

	<b>Initial guess</b>	<b>SACC</b>	<b>NSCC</b>	<b>OSS</b>
Cost	0.577	0.048	0.049	2.644
Step length (m)	0.2	1.10	1.0	1.0

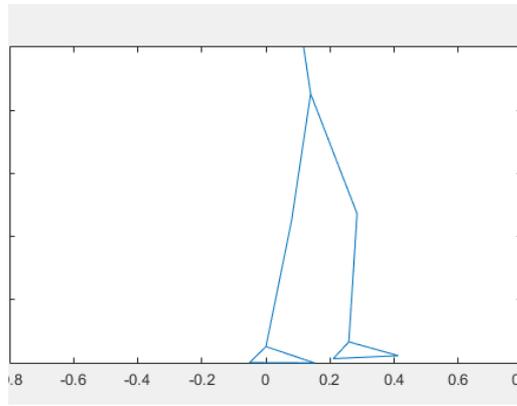
# Simulation Result (2/2)

$$\begin{aligned} \min_X \quad & \sum (q_k - q_{k_{ref}})^T (q_k - q_{k_{ref}}) \\ \text{s.t.} \quad & toe_{height} \geq f(toe_{horizontal}) \\ & heel_{height} \geq f(heel_{horizontal}) \\ & \text{Hermite-Simpson constraints} \end{aligned}$$



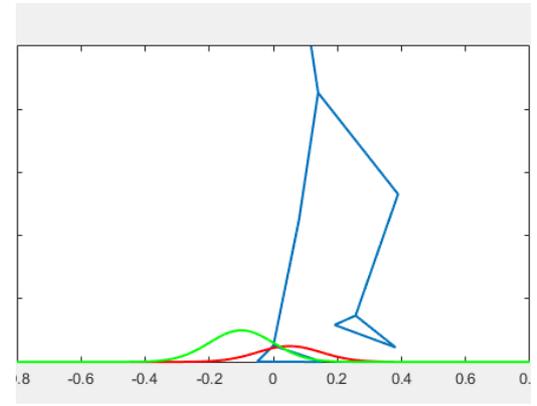
Initial guess

$Cost \approx 0.5$



Optimization through  
contact with SACC

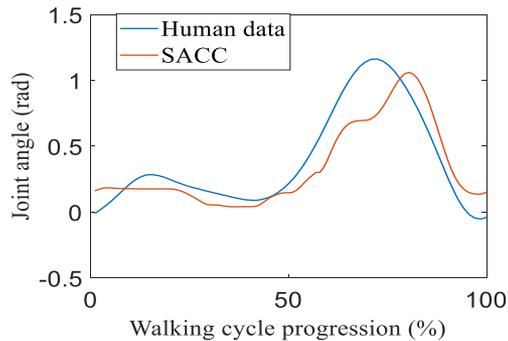
$Cost \approx 0.047$



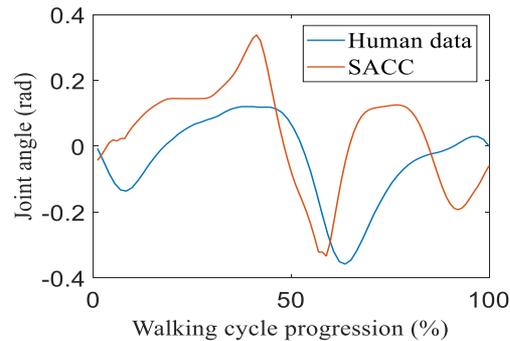
Modified result from a  
kinematics-based  
optimization

# Joint trajectories comparison to human data

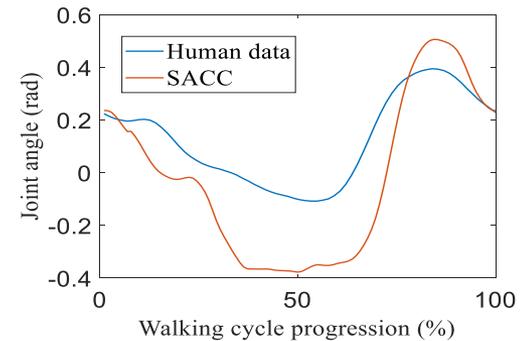
- Result from Opt. with SACC and post-processing Opt.



Knee joint

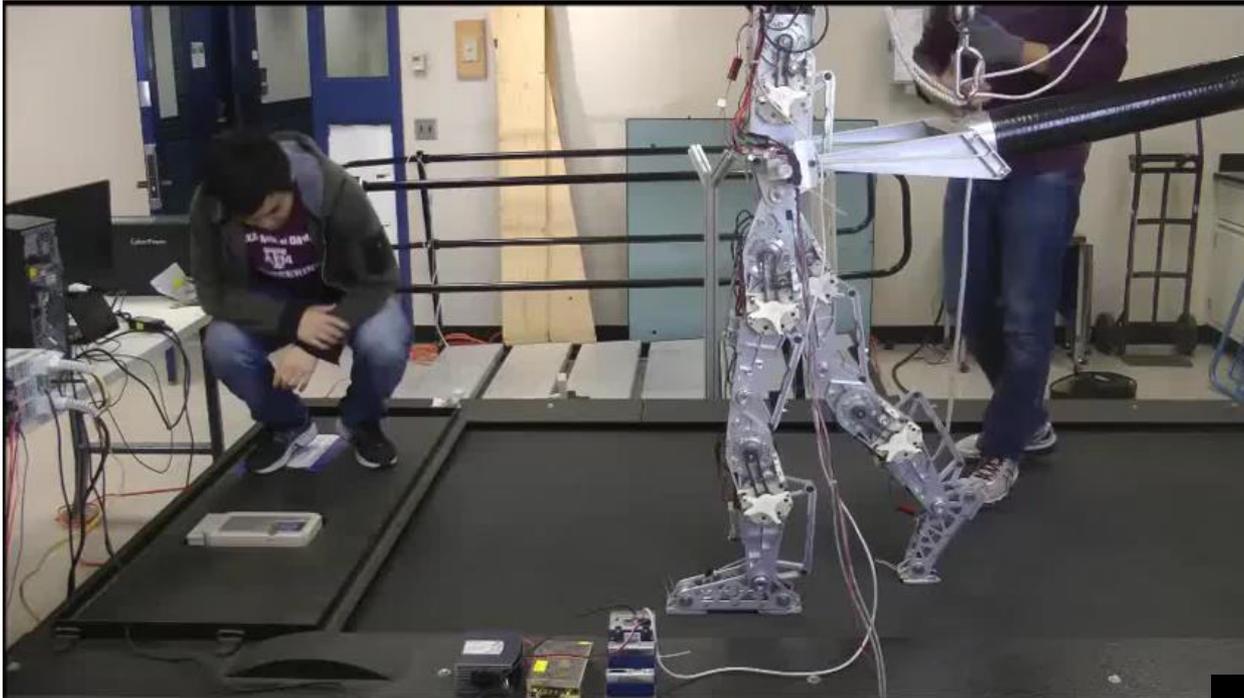


Ankle joint



Hip joint

# Current Experiment Result (Not completely successful)



# Conclusion and Future works

- The modified framework increases the accuracy of kinematic and dynamic approximation, and several schemes are provided and tested altering the walking behavior
- Further modifications of model and constraints (e.g. for increasing foot clearance) are required for more natural generated walking gait
- A better opt solver for handling optimization with complementary constraints and its relaxations is required
- Implementations on bipedal robot and lower limb prosthesis as validation

Thank you for your attention!

- Q&A