Unification of Locomotion Pattern Generation and Control Lyapunov Function-Based Quadratic Programs

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Introduction: Quadratic Program (QP)-based Control for bipedal walking

- In our application, QPs with affine constraints are solvable in real-time

- The structure of quadratic program is well suited to handle a diverse set of problems in robotic walking

QP-based Controller Examples for ZMP-based Walking

High-Level Controller

- Simplified model
- Center of Mass (COM) trajectory generation
- ZMP tracking

Model predictive control (MPC)*

COM trajectory

Low-Level Controller

- Full dynamics
- Torque control
- ZMP-based walking

QP-based controller with Control Lyapunov Functions (QP-CLFs)**

*P. B. Wieber, Int. Conf. Humanoid Robot, 2006.

** A. D. Ames and M. Powell, in *Control of Cyber-Physical Systems*, vol. 449, Springer.

Walking Control Problem Breakdown

Reasons:

- The complexity of the original walking control problem is high (e.g. control the actuator torque for tracking the desired ZMP)
- Real-time controller development

Cost behind task breakdown and cascade structure:

- Issues from the controller setup for both low-level control and high-level control

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Main Idea: A Unified Controller through Single Quadratic Program

Unified Quadratic Program

High-Level Controller

- Center of Mass (COM) trajectory generation
- ZMP tracking

Low-Level Controller

- Torque control
- ZMP-based walking

Outline

1) Bipedal Walking Control with Quadratic Program (QP) and ZMP Constraint
    a. ZMP and ZMP constraint
    b. Low-level control – ZMP-based walking control
    c. High-level control – COM pattern generation

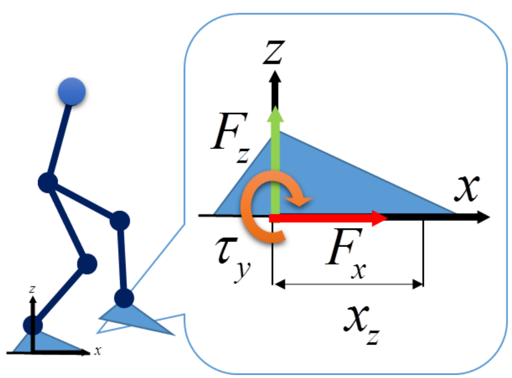
2) Unification of Walking Control and Pattern Generation
    a. QP setups of low-level and high-level controller
    b. Framework of the unified quadratic program

3) Result, Conclusion and Future work

Zero-Moment Point and Ground Reaction Forces

- Zero-moment point (ZMP), or equivalently center of pressure (COP), is the average of the pressure distribution:

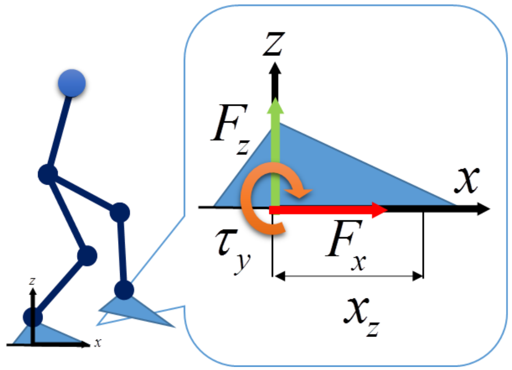
$$x_z = \frac{\int x F_z(x) dx}{\int F_z(x) dx} \qquad x_z : ZMP$$



Zero-Moment Point and Ground Reaction Forces

- ZMP can be expressed with ground reaction forces (GRFs)

$$x_z = -\frac{\tau_y}{F_z} \qquad x_z : ZMP$$

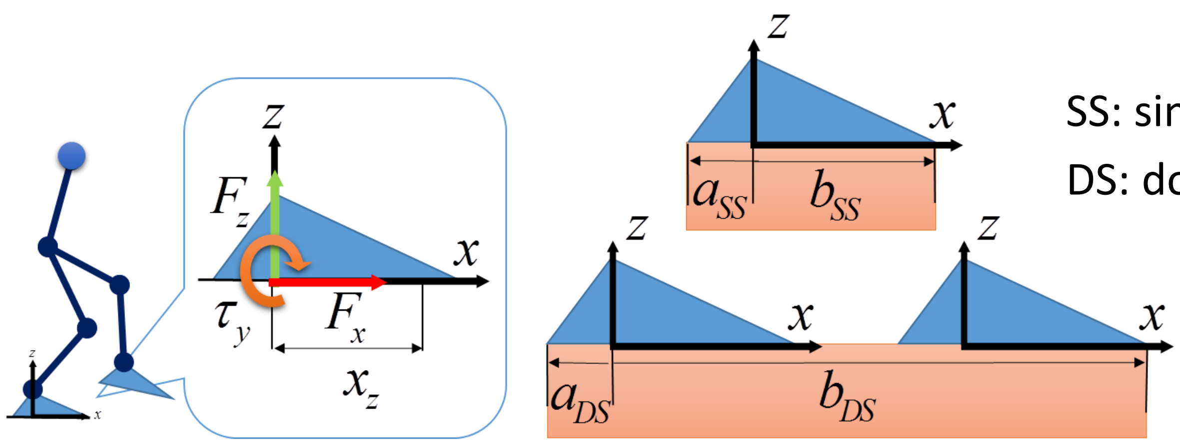


ZMP Constraint for Dynamic Balance

The legged system with footpad will not get tipping if its ZMP is inside its base of support (BOS) (or support polygon):

ZMP Constraint for Dynamic Balance

$$a_{\square} \leq x_z \leq b_{\square} \qquad x_z : ZMP$$



SS: single support

DS: double support

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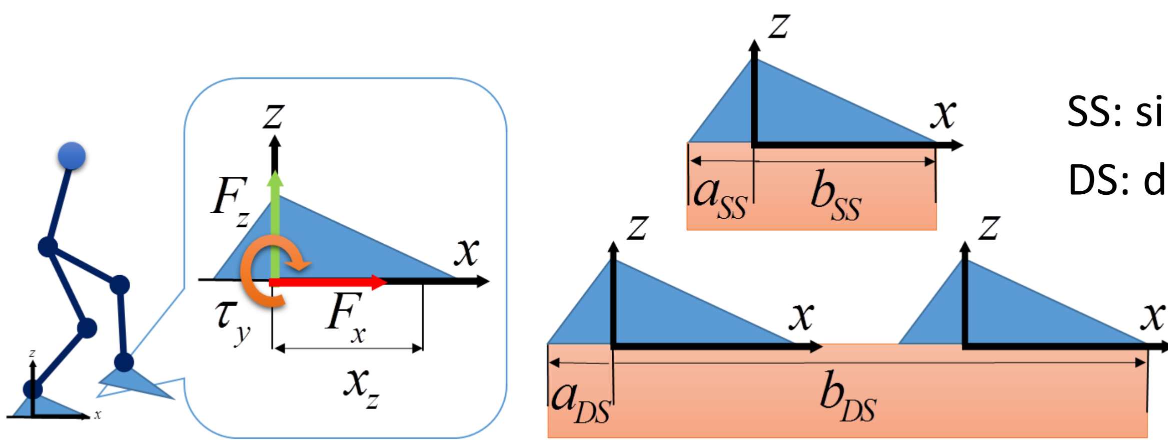

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ZMP Constraints and QP-based Controllers

Quadratic
Programs
with
ZMP Constraints

Foot placement
from
decision maker

High-Level Controller

- Center of Mass (COM) trajectory generation
- ZMP tracking

Low-Level Controller

- Torque control
- ZMP-based walking

Nonlinear Robot Control System with ZMP Constraints

QP-based controller with Rapidly Exponentially Stabilizing Control Lyapunov Function (RES-CLF)

- Control objectives (Control outputs/Virtual constraints) are given: For ZMP-based walking

- Nonlinear full constrained dynamics

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \begin{bmatrix} B & J_h^T \end{bmatrix} \begin{bmatrix} u \\ F \end{bmatrix} \triangleq \bar{B}(q)\bar{u}$$

u : the set of actuator torques
F : GRFs

- Requirement of constraints: Functions are affine in $\bar{u}$

Low-level controller

*A. D. Ames et.al ., IEEE Trans. Automat. Contr., 2014.

Nonlinear Robot Control System with ZMP Constraints

Formulation of QP-based controller with RES-CLF:

$$\begin{aligned} \bar{u}^* = \underset{u}{\operatorname{argmin}} \quad & \bar{u}^T H_{CLF} \bar{u} + f_{CLF}^T \bar{u} \\ \text{s.t.} \quad & \dot{V}_\varepsilon(x) \leq -\varepsilon V_\varepsilon(x) \qquad V_\varepsilon(x) : \text{RES-CLF} \\ & -bF_z \leq \tau_y \leq -aF_z \qquad x = [q, \dot{q}]^T \end{aligned}$$

Remarks

- Guaranteed ***instantaneous*** dynamic balance
- Guaranteed Lyapunov stability
- Potential issue

Low-level controller

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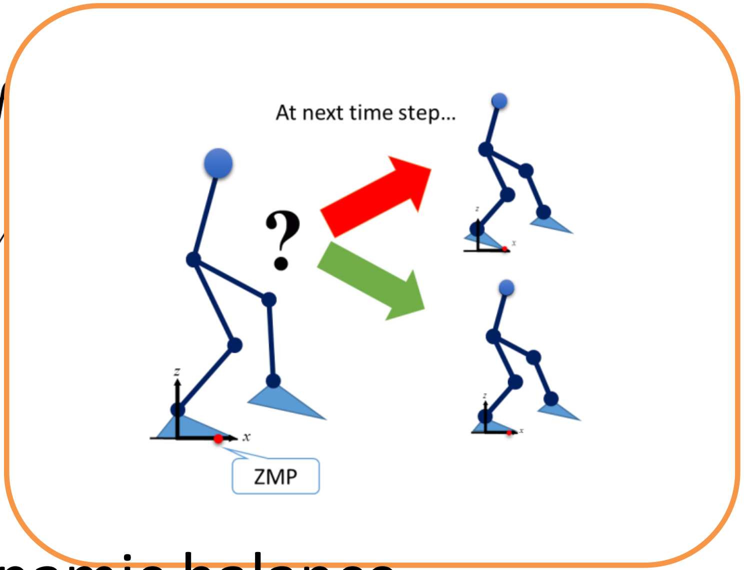
Nonlinear Robot Control System with ZMP Constraints

Formulation of QP-based controller with RES-CLF:

$$\bar{u}^* = \underset{u}{\operatorname{argmin}} \quad \bar{u}^T H_{CLF} \bar{u} + f$$

$$\text{s.t.} \quad \dot{V}_\varepsilon(x) \leq -\varepsilon V$$

$$-bF_z \leq \tau_y \leq$$



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Linear Inverted Pendulum Model for COM Trajectory Generation

Model Predictive Control (MPC) with Linear Inverted Pendulum (LIP) Model

- Objective: COM Trajectory Generation for tracking desired ZMP
- Equation of motion of LIP Model:

$$\ddot{x}_c = \frac{g}{z_0}(x_c - x_z) \triangleq \omega^2 (x_c - x_z)$$

g : Gravity constant
x_z : ZMP
x_c : COM
z_0 : Constant COM height

- Building block: Discretized state-space equation

$$x_{t+1} = \begin{bmatrix} 1 & \Delta T & 0 \\ \omega^2 \Delta T & 1 & -\omega^2 \Delta T \\ 0 & 0 & 1 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ 0 \\ \Delta T \end{bmatrix} u_t \triangleq A_t x_t + B_t u_t$$

$x_t = \begin{bmatrix} x_{ct} & \dot{x}_{ct} & x_{zt} \end{bmatrix}^T$
$u_t = \dot{z}_t$
ΔT : Sampling time

High-level controller

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  Equation of motion of LIP Model:

$$\ddot{x}_c = \frac{g}{z_0}(x_c - x_z) \triangleq \omega^2 (x_c - x_z)$$

- Open-loop predicted sequence for next N time-step (the horizon):

$$\bar{X} = \bar{A} X_{t_0} + \bar{B}\bar{U}$$

$$\bar{A} = \begin{bmatrix} A_t & A^2{}_t & \dots & A^{N-1}{}_t & A^N{}_t \end{bmatrix}^T$$

$$\overline{B} = \begin{bmatrix} B_t & 0 & \dots & \dots & 0 \\ A_t B_t & B_t & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ A_t{}^{N-2} B_t & A_t{}^{N-3} B_t & \dots & B_t & 0 \\ A_t{}^{N-1} B_t & A_t{}^{N-2} B_t & \dots & A_t B_t & B_t \end{bmatrix}$$

$$\bar{X} = \begin{bmatrix} x_{t+1} & \dots & x_{t+N} \end{bmatrix}^T$$

$$\bar{U} = \begin{bmatrix} u_{t+1} & \dots & u_{t+N} \end{bmatrix}^T$$

$$X_{t0} = \begin{bmatrix} x_{t_0} & \dots & x_{t_0} \end{bmatrix}^T$$

High-level controller

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Linear Inverted Pendulum Model for COM Trajectory Generation

Model Predictive Control (MPC) with LIP Model

QP formulation:

$$\bar{U}^* = \underset{\bar{U}}{\operatorname{argmin}} \quad \bar{U}^T H_p \bar{U} + f_p^T \bar{U}$$
$$\text{s.t.} \quad A_{iq,p}\bar{U} \leq b_{iq,p}$$

Remarks

- Guaranteed dynamic balance over the horizon
- Predictive ability improves the control performance
- Potential Issues

High-level controller

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Conventional Setup: Cascade Control

Foot placement from decision maker

High-Level Controller

- Center of Mass (COM) trajectory generation
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ZMP Constraints

Low-Level Controller

- Torque control
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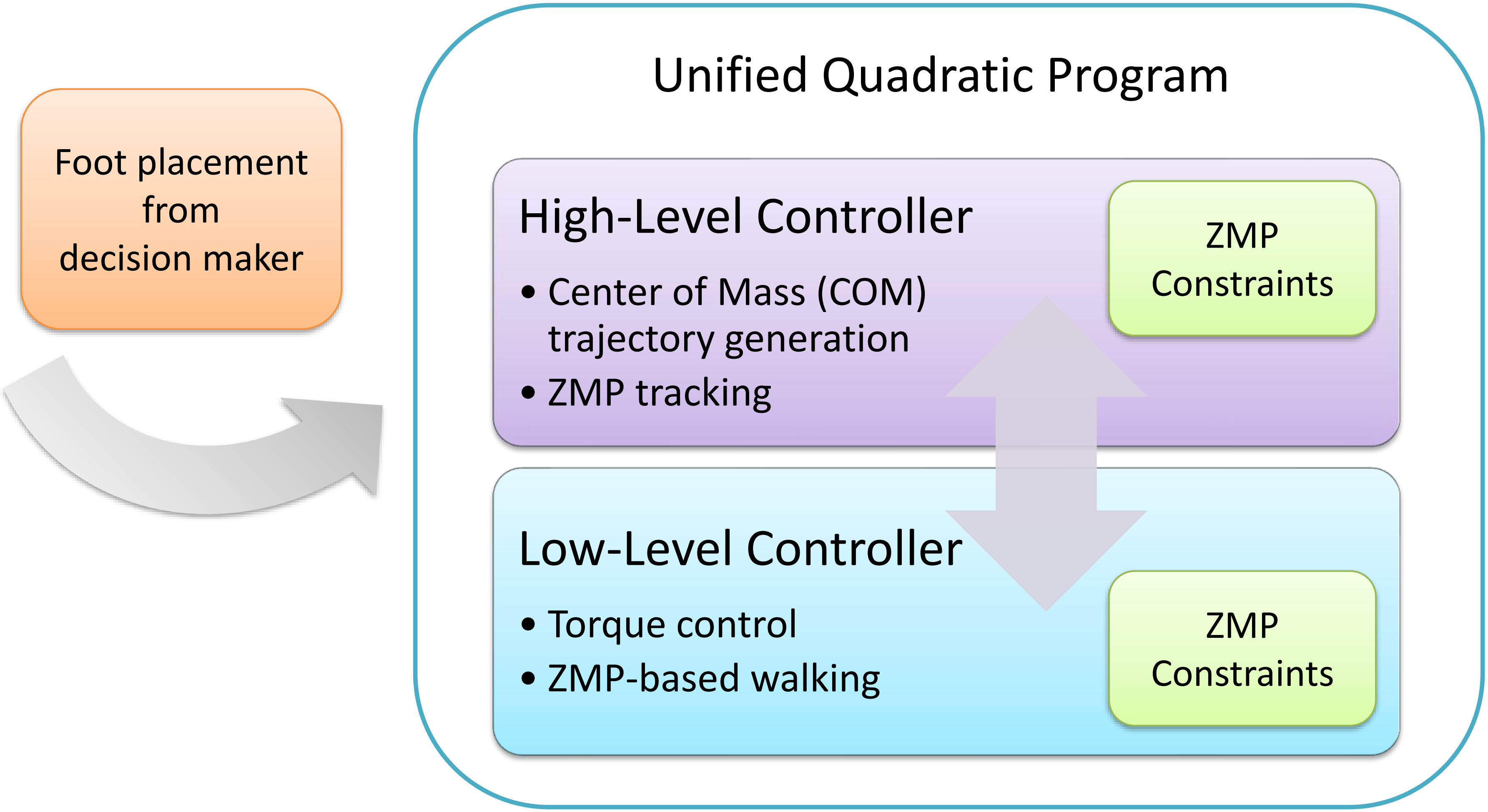
ZMP Constraints

Main Approach: A Unified Controller through Quadratic Program



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3) Results and Future work

The Setup for CLF-QP: The Construction of RES-CLF

General nonlinear system form:

$$\dot{x} = f(x) + g(x)\bar{u}$$

Control outputs:

- Swing foot position and orientation, torso angle
- Center of mass position x_c

$$y(q) \triangleq y_a(q) - y_d(t) \Rightarrow y \to 0$$

Input/output relation

$$\ddot{y} = L_f^2 y(x) + L_f L_g y(x)\bar{u} + \ddot{y}_d \triangleq L_f + \bar{A}\bar{u} + \ddot{y}_d$$

Desired output dynamics by feedback linearization

$$\bar{u} = \bar{A}^{-1}(-L_f + \mu + \ddot{y}_d) \Rightarrow \ddot{y} = \mu$$

Dynamics of the linearized system

$$\dot{\eta} = F\eta + G\mu \qquad \eta = [y, \dot{y}]^T$$

RES - Control Lyapunov Function

$$V_\varepsilon(\eta) = \eta^T P_\varepsilon \eta \qquad P_\varepsilon = \begin{bmatrix} \varepsilon I & 0 \\ 0 & I \end{bmatrix} P \begin{bmatrix} \varepsilon I & 0 \\ 0 & I \end{bmatrix}$$

$$F^T P + PF - PGG^T P + Q = 0,$$
$$\text{where } Q = Q^T > 0,\ P = P^T > 0$$

RES-CLF Constraint with Relaxation:

$$\dot{V}_\varepsilon(\eta) = L_f V_\varepsilon(\eta) + L_g V_\varepsilon(\eta)\mu \le -\varepsilon V_\varepsilon(\eta) + \delta$$

Low-level controller

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$$\dot{V}_\varepsilon(\eta) = L_f V_\varepsilon(\eta) + L_g V_\varepsilon(\eta)\mu \leq \boxed{-\varepsilon V_\varepsilon(\eta) + \delta}$$

Low-level controller

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The Setup for CLF-QP: The QP Formulation

Cost function: Minimize μ and panelize δ

Important constraints:

- ZMP constraint $a_{\square} \le -\tau_y / F_z \le b_{\square}$
- CLF constraint $\dot{V}_{\varepsilon}(\eta) = L_f V_{\varepsilon}(\eta) + L_g V_{\varepsilon}(\eta)\mu \le -\varepsilon V_{\varepsilon}(\eta) + \delta$
- torque constraint $u_{min} \le u \le u_{max}$
- normal force constraint $F_z \ge 0$

CLF-QP:

$$\bar{u}^* = \underset{\bar{u},\delta}{\operatorname{argmin}} \quad \bar{u}^T H_{CLF} \bar{u} + f_{CLF}^T \bar{u} + p\delta^2$$

$$\text{s.t.} \quad A_{iq,CLF} \begin{bmatrix} \bar{u} \\ \delta \end{bmatrix} \le b_{iq,CLF}$$

$$A_{eq,CLF} \begin{bmatrix} \bar{u} \\ \delta \end{bmatrix} = b_{eq,CLF}$$

Expression of COM acceleration via Lie derivative

$$\ddot{x}_c = L_f^2 x_c + L_f L_g x_c \bar{u}$$

Low-level controller

The Setup for MPC-QP: The ZMP Constraint

General setup:

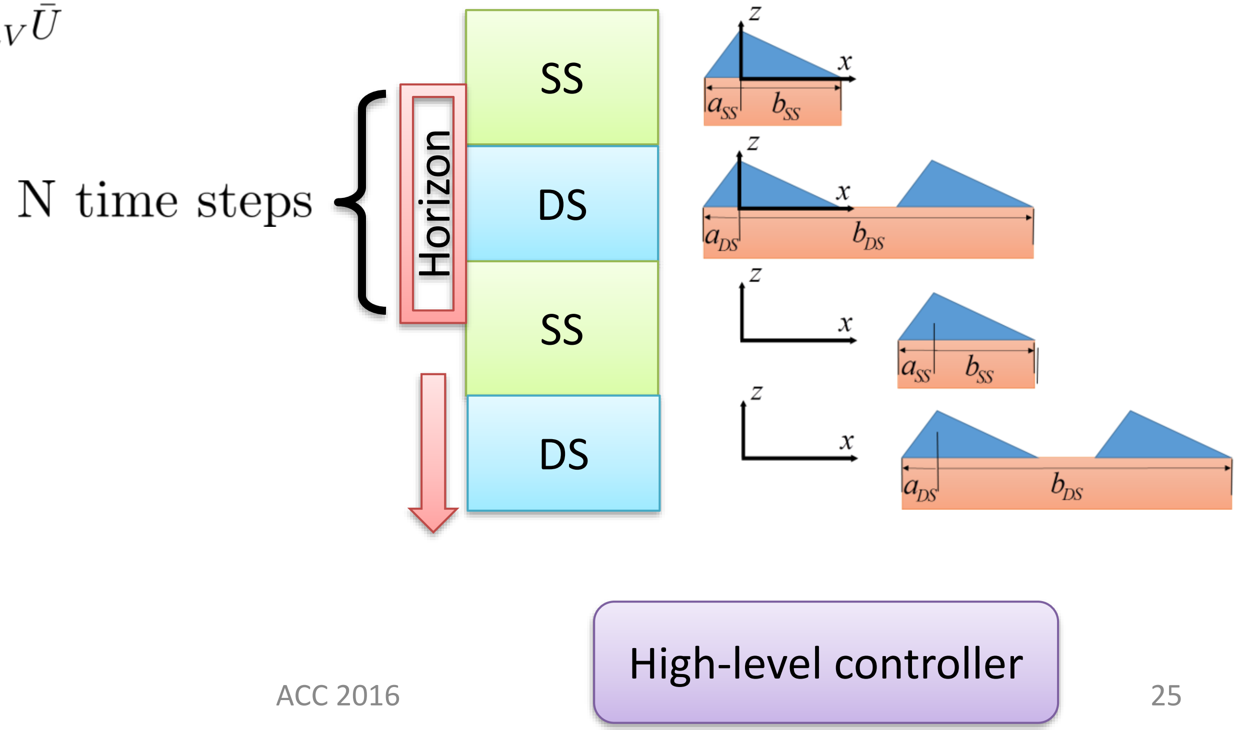
$$\bar{X} = \bar{A}X_{t_0} + \bar{B}\bar{U}$$

$$\begin{aligned} \bar{X}_z &= \bar{A}_{zmp}\bar{X}_{t_0} + \bar{B}_{zmp}\bar{U} \\ \bar{X}_c &= \bar{A}_{com}\bar{X}_{t_0} + \bar{B}_{com}\bar{U} \\ \dot{\bar{X}}_c &= \bar{A}_{comV}\bar{X}_{t_0} + \bar{B}_{comV}\bar{U} \end{aligned}$$

Horizon computation

- Horizon Length $N = N_{SS} + N_{DS}$

ZMP constraint $\bar{a} \leq \bar{X}_z \leq \bar{b}$



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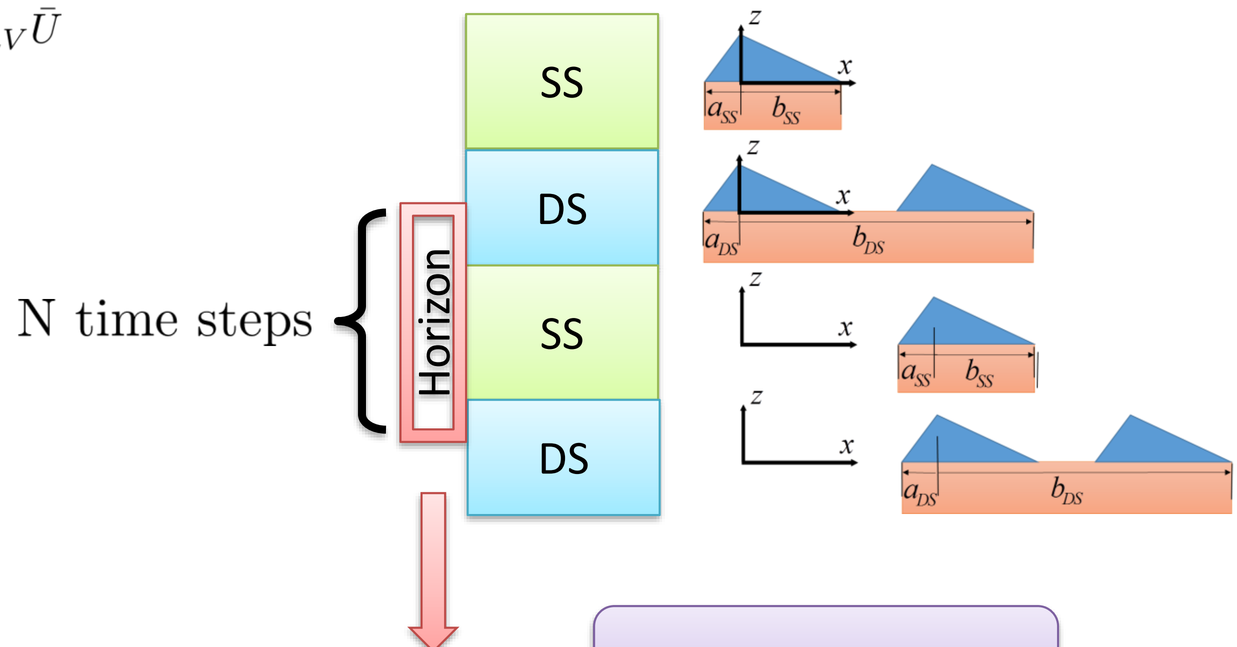

High-level controller

The Setup for MPC-QP: The QP Formulation

Cost function: $\omega_1 \bar{U}^T \bar{U} + \omega_2 |\bar{X}_z - \bar{X}_z^{goal}|^2$

Important constraints:

- ZMP constraint $\bar{a} \leq \bar{X}_z \leq \bar{b}$
- Terminal constraints $x_{c_{t_0+N}} = x_c^{goal}$

$$\dot{x}_{c_{t_0+N}} = \dot{x}_c^{goal}$$

MPC-QP:

$$\begin{aligned} \underset{\bar{U}^*}{\text{argmin}} \quad & \tfrac{1}{2}\bar{U}^T H_p \bar{U} + {f_p}^T \bar{U} \\ \text{s.t.} \quad & A_{eq,p}\bar{U} = b_{eq,p} \\ & A_{iq,p}\bar{U} \leq b_{iq,p} \end{aligned}$$

Equation of motion of LIP model

$$\ddot{x}_c = \frac{g}{z_0}(x_c - x_z) \triangleq \omega^2 (x_c - x_z)$$

High-level controller

The Main Result: The Unified Quadratic Program

$$\underset{\bar{u}^*,\bar{U}^*,\delta^*}{\operatorname{argmin}} \frac{1}{2} \begin{bmatrix} \bar{u} \\ \bar{U} \\ \delta \end{bmatrix}^T \begin{bmatrix} H_{CLF} & 0 & 0 \\ 0 & H_p & 0 \\ 0 & 0 & p \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{U} \\ \delta \end{bmatrix} + \begin{bmatrix} f_{CLF} \\ f_p \\ 0 \end{bmatrix}^T \begin{bmatrix} \bar{u} \\ \bar{U} \\ \delta \end{bmatrix}$$

$$\text{s.t.} \quad \begin{bmatrix} A_{eq,CLF} & 0 & 0 \\ 0 & A_{eq,p} & 0 \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{U} \\ \delta \end{bmatrix} = \begin{bmatrix} b_{eq,CLF} \\ b_{eq,p} \end{bmatrix}$$

$$\begin{bmatrix} A_{iq,CLF} & 0 & -1 \\ 0 & A_{iq,p} & 0 \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{U} \\ \delta \end{bmatrix} \leq \begin{bmatrix} b_{iq,CLF} \\ b_{iq,p} \end{bmatrix}$$

$$(L_f^2 x_c + L_f L_g x_c \bar{u}) \frac{z_0}{g} - x_c = -x_z$$

The Main Result: The Unified Quadratic Program

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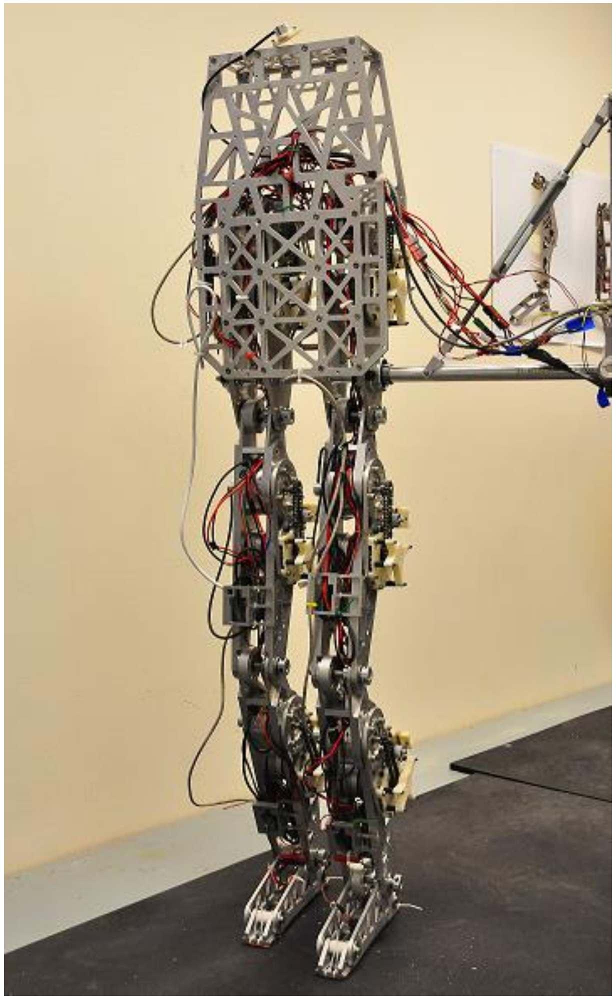
$$\ddot{x}_c = L_f^2 x_c + L_f L_g x_c \bar{u}$$

$$(L_f^2 x_c + L_f L_g x_c \bar{u}) \frac{z_0}{g} - x_c = -x_z$$

$$\ddot{x}_c = \frac{g}{z_0}(x_c - x_z) \triangleq \omega^2 (x_c - x_z)$$

Results

- Planer Bipedal Robot Amber 3
  - Height: 1.45 m
  - Weight: 33.4 Kg
  - 7-link, 6 DOFs

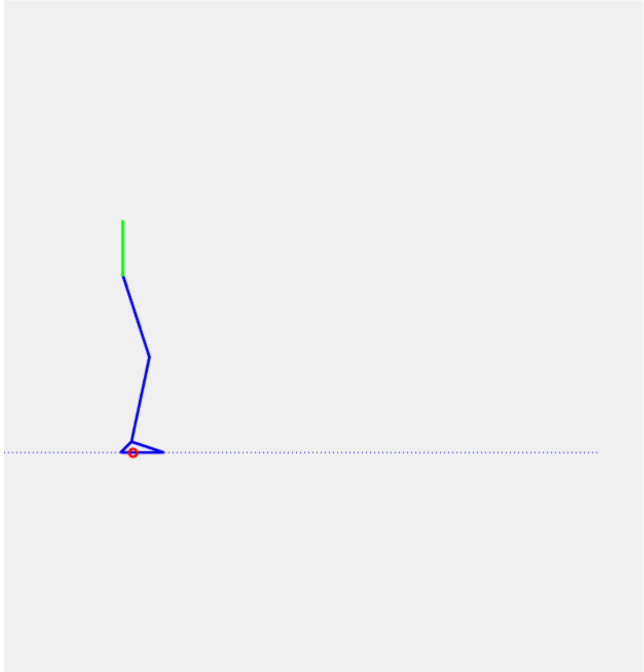


- Walking and Controller

Parameter	Value	Parameter	Value
T_{SS}	2 s	T_{DS}	1 s
MPC sampling time ΔT	0.1 s	Length of MPC horizon	3 s
L_{step}	10 cm	Stride Height	5 cm

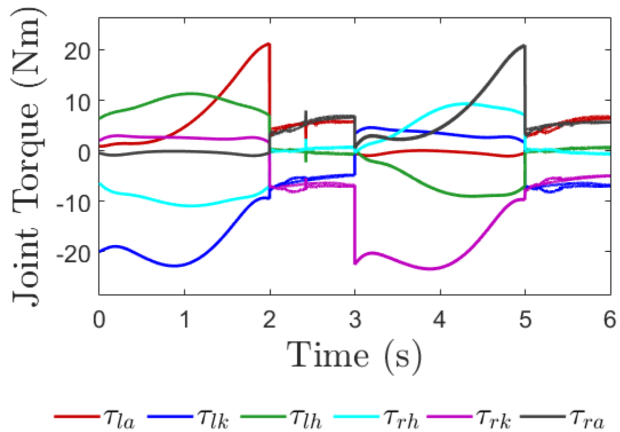
Adjustments for Unified QP

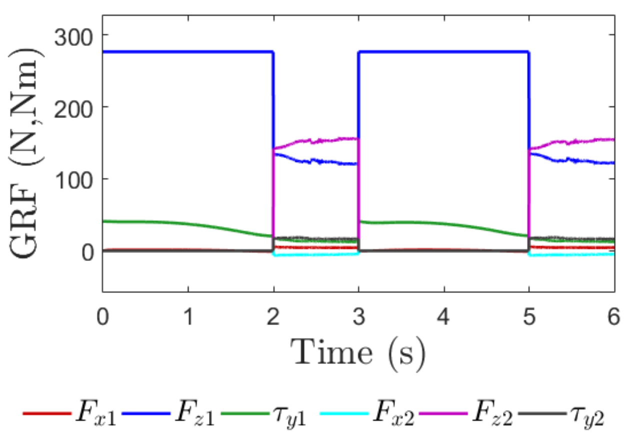
- COM was removed from the control outputs in CLF-QP
- The feedback of postimpact COM velocity to MPC-QP was set to zero to enforce the COM planned as free of impact
- Terminal constraints of COM and COM velocity in MPC-QP were removed



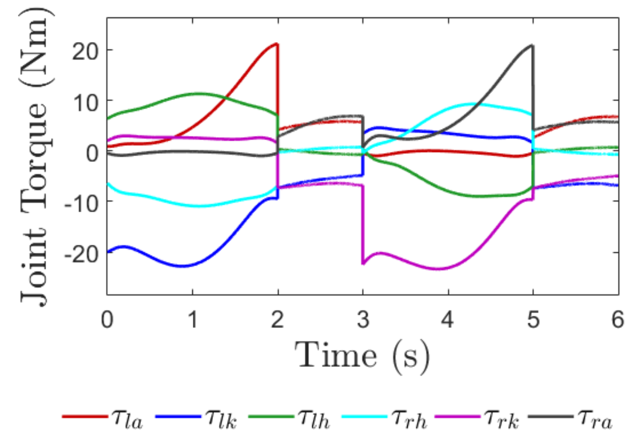
Settlements for Unified QP

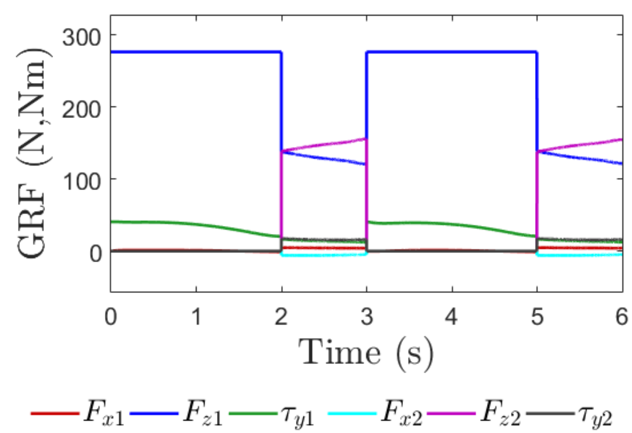
w/ COM terminal Constraints



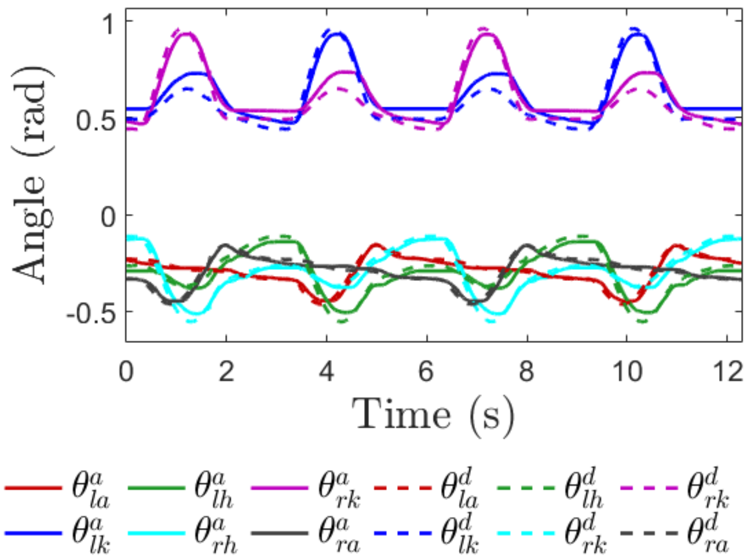


w/o COM terminal Constraints





Experiment



Angle (rad)
1
0.5
0
-0.5
0
2
4
6
8
10
12
Time (s)
θ^a_{la} θ^a_{lh} θ^a_{rk} θ^d_{la} θ^d_{lh} θ^d_{rk}
θ^a_{lk} θ^a_{rh} θ^a_{ra} θ^d_{lk} θ^d_{rk} θ^d_{ra}

Conclusion

- We proposed a novel method of combing real-time walking pattern generation and constrained nonlinear control under ZMP constraints and torque constraints.

$$\underset{\bar{u}^*,\bar{U}^*,\delta^*}{\operatorname{argmin}} \frac{1}{2}\begin{bmatrix}\bar{u}\\ \bar{U}\\ \delta\end{bmatrix}^T \begin{bmatrix}H_{CLF} & 0 & 0\\ 0 & H_p & 0\\ 0 & 0 & p\end{bmatrix}\begin{bmatrix}\bar{u}\\ \bar{U}\\ \delta\end{bmatrix} + \begin{bmatrix}f_{CLF}\\ f_p\\ 0\end{bmatrix}^T \begin{bmatrix}\bar{u}\\ \bar{U}\\ \delta\end{bmatrix}$$

$$\text{s.t.} \quad \begin{bmatrix}A_{eq,CLF} & 0 & 0\\ 0 & A_{eq,p} & 0\end{bmatrix}\begin{bmatrix}\bar{u}\\ \bar{U}\\ \delta\end{bmatrix} = \begin{bmatrix}b_{eq,CLF}\\ b_{eq,p}\end{bmatrix}$$

$$\begin{bmatrix}A_{iq,CLF} & 0 & -1\\ 0 & A_{iq,p} & 0\end{bmatrix}\begin{bmatrix}\bar{u}\\ \bar{U}\\ \delta\end{bmatrix} \leq \begin{bmatrix}b_{iq,CLF}\\ b_{iq,p}\end{bmatrix}$$

$$(L_f^2 x_c + L_f L_g x_c \bar{u})\frac{z_0}{g} - x_c = -x_z$$

- The unified QP have advantages of both QPs: it resolves control actions which locally stabilize nonlinear control system outputs while ensuring that these control actions are consistent with a forward horizon COM plan that satisfies ZMP constraints in the simplified model.

Future work

- Completing a real-time implementation of the unified QP controller in C++.
- Robustness tests: push recovery or walking through uneven terrain are planned to be conducted.
- Further generalization and unification, such as combing footstep planning, manipulation, or time parameterization for event-based locomotion are also considered.

Thank you for your attention!

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Q & A ?

ZMP Constraints and Controller Overview

Low-level controller: QP-CLF

- Pros
  - Real-time implementations
  - Minimized control effort
  - Exploiting nonlinear full dynamics
- Cons
  - ZMP constraint only for current time-step

High-level controller: MPC

- Pros
  - Real-time implementation
  - Optimal choice considering the future prediction
  - ZMP constraint over the whole horizon
- Cons
  - Control input sequence may not be feasible
  - Simplified model may not reflect the real dynamics