Minimizing Angular Momentum Yields More Robust Walking Trajectories in Five-Link Biped

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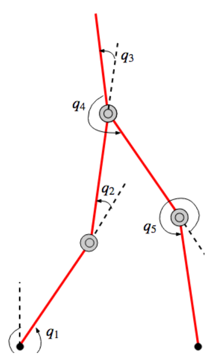
Introduction

In the field of humanoid robots, walking motions are typically achieved by calculating desired trajectories and using controllers to help the robot follow those trajectories. The manner in which these trajectories are obtained is critical, since even the best controllers cannot yield a stable walking gait if the trajectories which they track are of poor quality.

In this study, a trajectory optimization via direct collocation is performed to generate desired joint trajectories for a single step of a five-link biped. The trajectories are examined in a forward simulation for their robustness to impulsive disturbances. Prior research has shown that angular momentum H about the whole body center of mass (COM) is highly regulated during human walking [1]. Since human walking is seemingly robust to disturbances and has very low H throughout the gait cycle, it is reasonable to suspect that trajectories with reduced H would be more robust to disturbances than those without. Thus, terms which facilitate the minimization of H are added to the cost function, and the performance of base-line trajectories is compared to the performance of trajectories which also minimize H. It is hypothesized that the trajectories which minimize angular momentum will be more robust to disturbances.

Methods

The simulated walking robot is the five-link biped shown in Figure 1(a). It has dimensions, masses, and inertias similar to that of a human for the shank and thigh, and uses the head-arms-torso (HAT) approximation for the upper body [2].



$$
\begin{aligned}
&\min \sum \tau^2 + \alpha H_{\text{spin}}^2 + \beta H_{\text{orb}}^2 \\
&s.t. \\
&\text{Dynamics} = 0 \\
&\text{ImpactDynamics} = 0 \\
&\text{Periodicity} = 0 \\
&\text{JointRange} \le 0 \\
&\text{TorqueLimit} \le 0 \\
&\text{StepLength} \le 0 \\
&\text{ImpactVelocity} \le 0
\end{aligned}
$$

Figure 1: (a) Five link biped **(b)** Optimization formulation. Note that all inequality constraints are expressed with "≤ 0".

The joint trajectories are generated via trajectory optimization using direct collocation, in which the desired optimal trajectories are discretized and used as the decision variables in the optimization [3]. In this case, the accelerations, post-impact velocities, and impact forces are included with the decision variables in addition to those shown in [3]. The optimization is done in Julia (v1.3) using JuMP as an interface and IPOPT as the solver. The Hermite-Spimpson method is used for the direct collocation [3]. The base-line cost function is the sum of the squared joint torques τ^2. This cost function is augmented with the spin angular momentum squared H_{spin}^2 (i.e., $\alpha = 1, \beta = 0$), with the orbital angular momentum squared H_{orb}^2 (i.e., $\alpha = 0, \beta = 1$), or with both H_{orb}^2 and H_{spin}^2 (i.e., the total angular momentum squared, $\alpha = 1, \beta = 1$). H_{orb} is the angular momentum of a segment COM (point mass) in the reference frame of the whole-body COM. H_{spin} is the angular momentum of a segment about its own COM (in its own reference frame). The optimization includes many constraints such as those for respecting the (Euler-Lagrange) dynamics, a minimum step length of 0.2 m, periodicity constraints so that the step behavior is repeatable, limits on the joint angles, and impact at the heel strike. A formulation of the optimization with these constraints is given in Figure 1(b). The details of these constraints can be found in [4].

A forward simulation is conducted in which a horizontal impulsive force is applied at 1.0 sec at the hip joint for a duration of approximately 0.4 sec, against the direction of the step. The biped is tasked with walking 10 steps without falling over. A PD controller is used for all simulations to track the trajectories, and center of mass forward progression is used as the phase variable (i.e., time parameterized by state variables).

Results and Discussion

The maximum force the biped withstands without falling over for a given cost function's set of optimal trajectories is reported in Table 1. For the first row of forces, the step length was left unconstrained except for the minimum required length of 0.2 m. To rule out the potential effects of step lengths on the maximum force, trajectories were generated for fixed step length as well. Those numbers are given in the bottom three rows. Bold numbers indicate that the fixed step length is the same as the step length chosen by the optimizer in the first row.

Table 1: Maximum force (N) of perturbation without falling for sets of trajectories with different cost functions and step lengths.

Step Length [m]	τ^2	τ^2+H_{spin}^2	τ^2+ H_{orb}^2	τ^2+H_{spin}^2+ H_{orb}^2
minimum 0.200	222	494	536	661
0.200	**222**	488	**536**	631
0.241	221	499	563	**661**
0.272	unstable	**494**	570	666

Adding angular momentum into the cost function improved the maximum force tolerated in all cases. Including H_{orb}^2 alone was better than including H_{spin}^2 alone, but including both resulted in the greatest force tolerated without falling.

Significance

These results support the idea that human walking tends to have low angular momentum because it allows human gait to be more robust to disturbances.

References

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[2] Winter, *Wiley*, 4th ed., 2009
[3] Kelly, *SIAM Review*, 59(4), pp849-904, 2017
[4] Chao et al., *IROS*, pp1435-1440, 2019