Optimal Bipedal Walking Gaits Found with Different Direct Collocation Settings

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Introduction

Applications of bipedal robotics include the navigation of uneven terrain [1] and the development of assistive devices like prostheses [2]. These devices may require an optimal reference trajectory. In order to find such a continuous-time gait, it is helpful to transcribe this problem into a discretized nonlinear program (NLP). One technique, direct collocation, does this by satisfying the system's dynamics at only a few discrete points along the trajectory [3].

This method can be implemented in many different ways. Some studies have compared the specific collocation method chosen in terms of time or accuracy [4], and some have compared differentiation methods in a variety of contexts, including other forms of trajectory optimization [5]. However, such studies have not been completed for the low degree-of-freedom (DOF) bipedal walking models sometimes used by roboticists, such as the two-DOF compass walker and the five-DOF kneed biped with torso. This study [6] addresses this gap in the literature and compares factors such as CPU time for optimal gaits generated with different implementation settings.

Methods

NLPs were developed for finding an optimal trajectory for the two walkers mentioned above. For both models, the objective function was the integral of the sum of squared input torques. Constraints included periodicity constraints, constraints on the initial orientations of the models' legs, and dynamic constraints. For the five-link walker, constraints were also used to prevent the hyperextension of the knees and to ensure foot clearance.

Trapezoidal (TPZD) and Hermite-Simpson (H-S) collocation were used in the objective function and collocation constraints of the compass walker, while only the former was used with the five-link biped. The gradient of the objective and Jacobian of the constraints for the compass walker were found either numerically (ND)—using the central difference approximation—or symbolically (SD). For the five-link walker, these were found using ND, except for the Jacobian of the collocation constraints, which varied in method. Also, for this walker, the joint accelerations were either found numerically or symbolically, or they were included in the decision variables.

Optimizations were performed using the mexIPOPT [7] package for MATLAB. One-hundred optimizations for each of four settings were run for the compass walker with random initial guesses and eleven collocation points. Twenty optimizations were run for each of four settings of the five-link walker. The data were analyzed using an analysis of variance (ANOVA).

Results and Discussion

Figure 1(a) shows the mean and sample standard deviation of two time-related measures for the compass walker. ND was significantly faster than SD, so this differentiation method was considered to be ideal. TPZD was selected over H-S, also for its speed, although there was a slight tradeoff with accuracy.

Similar types of data are shown for the five-link walker in Figure 1(b). It was recommended that accelerations be included in the decision variables due to the rapid per-iteration CPU time

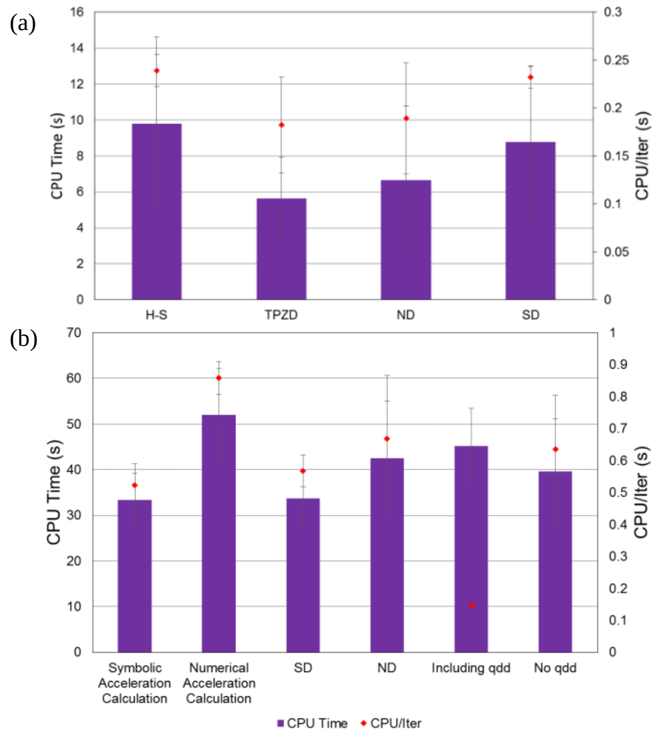

Figure 1: Total CPU time and average CPU time per iteration for (a) compass walker, (b) five-link walker with 21 collocation points. [6]

and a suggested robustness. This was at the expense of the overall CPU time. It was also determined that the joint accelerations should be calculated symbolically due to speed and that ND should be used due to the high complexity of the symbolic expressions required for SD.

In summary, varying the implementation of a direct collocation-based walking trajectory optimization resulted in changes in solution efficiency and, in some cases, accuracy. By analyzing tradeoffs, it was possible to determine potential best practices for performing such optimizations.

Significance

Since many applications of bipedal robotics, such as powered assistive devices, may require a reference trajectory, these results could help to expedite the requisite optimizations. On a wider scale, this study could be combined with those in other fields to see if best practices are similar for multiple types of NLPs with collocation methods. That would extend the value of these results to a far wider range of fields which require the solution of NLPs.

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